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## **The Accuracy of Extrapolation (Time Series) Methods: Results of a Forecasting Competition**

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### **ABSTRACT**

In the last few decades many methods have become available for forecasting. As always, when alternatives exist, choices need to be made so that an appropriate forecasting method can be selected and used for the specific situation being considered. This paper reports the results of a forecasting competition that provides information to facilitate such choice. Seven experts in each of the 24 methods forecasted up to 1001 series for six up to eighteen time horizons. The results of the competition are presented in this paper whose purpose is to provide empirical evidence about *differences* found to exist among the various extrapolative (time series) methods used in the competition.

**KEYWORDS** Forecasting Time series Evaluation Accuracy  
Comparison Empirical study

Forecasting is an essential activity both at the personal and organizational level. Forecasts can be obtained by:

- (a) purely judgemental approaches;
- (b) causal or explanatory (e.g. econometric or regression) methods;
- (c) extrapolative (time series) methods; and
- (d) any combination of the above.

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Furthermore, as there are many approaches or methods available within (a), (b), (c), there is considerable choice for selecting a single approach, method, or combination procedure to predict future events. The implications of making the right choice are extremely important both from a theoretical standpoint and in practical terms. In many situations even small improvements in forecasting accuracy can provide considerable savings.

It is important to understand that there is no such thing as the best approach or method as there is no such thing as the best car or best hi-fi system. Cars or hi-fis differ among themselves and are bought by people who have different needs and budgets. What is important, therefore, is not to look for 'winners' or 'losers', but rather to understand how various forecasting approaches and methods differ from each other and how information can be provided so that forecasting users can be able to make rational choices for their situation.

Empirical studies play an important role in better understanding the pros and cons of the various forecasting approaches or methods (they can be thought of as comparable to the tests conducted by consumer protection agencies when they measure the characteristics of various products).

In forecasting, accuracy is a major, although not the only factor (see note by Carbone in this issue of the *Journal of Forecasting*) that has been dealt with in the forecasting literature by empirical or experimental studies. Summaries of the results of published empirical studies dealing with accuracy can be found in Armstrong (1978), Makridakis and Hibon (1979), and Slovic (1972). The general conclusions from these three papers are: (a) Judgemental approaches are not necessarily more accurate than objective methods; (b) Causal or explanatory methods are not necessarily more accurate than extrapolative methods; and (c) More complex or statistically sophisticated methods are not necessarily more accurate than simpler methods.

The present paper is another empirical study concerned mainly with the *post-sample* forecasting accuracy of extrapolative (time series) methods. The study was organized as a 'forecasting competition' in which expert participants analysed and forecasted many real life time series.

This paper extends and enlarges the study by Makridakis and Hibon (1979). The major differences between the present and the previous study owe their origins to suggestions made during a discussion of the previous study at a meeting of the Royal Statistical Society (see Makridakis and Hibon, 1979) and in private communications. The differences are the following:

1. The number of time series used was increased from 111 to 1001 (because of time constraints, not all methods used all 1001 series).
2. Several additional methods were considered and, in some cases, different versions of the same method were compared.
3. Instead of a single person running all methods, experts in each field analysed and forecasted the time series.
4. The type of series (macro, micro, industry, demographic), time intervals between successive observations (monthly, quarterly, yearly) and the number of observations were recorded and used (see Table 1).
5. The time horizon of forecasting was increased (18 periods for monthly data, 8 for quarterly and 6 for yearly).
6. Initial values for exponential smoothing methods were obtained by 'back-forecasting'—a procedure common in the Box-Jenkins method.
7. Additional accuracy measures were obtained (notably mean square errors, average rankings and medians).

The paper is organized as follows: first, an estimate of the time needed and computer cost incurred for each method will be given; second, the data used are briefly described; third, summary measures of overall accuracy are given; fourth, the effects of sampling errors are discussed; what

would have happened had another set of series been selected?; fifth, differences among the various methods will be presented; sixth, the conditions under which various methods are better than others are discussed. Finally, an evaluation of the results and some general conclusions are presented. There will also be two appendices describing the accuracy measures and the methods used.

### TIME AND COST OF RUNNING THE VARIOUS METHODS

According to statements by the participants of the competition, the Box-Jenkins methodology (ARMA models) required the most time (on the average more than one hour per series). This time included looking at the graph of each series, its autocorrelation and partial autocorrelation functions, identifying an appropriate model, estimating its parameters and doing diagnostic checking on the residual autocorrelations. The method of Bayesian forecasting required about five minutes of personal time to decide on the model to be used and get the program started. Apart from that, the method was run mechanically.

All other methods were run on a completely automatic basis. That is, the various data series were put in the computer, and forecasts were obtained with no human interference. This means that the model selection (if needed) and parameter estimation were done automatically and that the forecasts were *not* modified afterwards through any kind of human intervention. All results can, therefore, be exactly replicated by passing the data through the program.

### THE DATA

The 1001 time series were selected on a quota basis. Although the sample is not random, in a statistical sense, efforts were made to select series covering a wide spectrum of possibilities. This included different sources of statistical data and different starting/ending dates. There were also data from firms, industries and nations. Table 1 shows the major classifications of the series. All

Table 1

| Time interval between successive observations | Types of time series data |                 |                       |          |                             |                                   |             |       |
|-----------------------------------------------|---------------------------|-----------------|-----------------------|----------|-----------------------------|-----------------------------------|-------------|-------|
|                                               | Micro-data                |                 |                       | Industry | Macro-data                  |                                   | Demographic | Total |
|                                               | Total firm                | Major divisions | Below major divisions |          | GNP or its major components | Below GNP or its major components |             |       |
| Yearly                                        | 16                        | 29              | 12                    | 35       | 30                          | 29                                | 30          | 181   |
| Quarterly                                     | 5                         | 21              | 16                    | 18       | 45                          | 59                                | 39          | 203   |
| Monthly                                       | 10                        | 89              | 104                   | 183      | 64                          | 92                                | 75          | 617   |
| Subtotal                                      | 31                        | 139             | 132                   | 236      | 139                         | 180                               | 144         | 1001  |
| TOTAL                                         | 302                       |                 |                       | 236      | 319                         |                                   | 144         | 1001  |

accuracy measures were computed, using these classifications. Unfortunately, the output is many thousands of pages long and can only be reported in this paper in a summary form. However, a computer tape containing the original series and the forecasts of each method, together with the programs used for the evaluation, can be obtained by writing to A. Andersen, R. Carbone or S. Makridakis (whoever is geographically closest), because a major ground rule for this competition has been that all of the results could be replicated by anyone interested in doing so. Also, interested readers can write to S. Makridakis to obtain more or all of the results, or they may wish to wait until a book (Makridakis *et al.*, 1983) describing the methods and the study in detail is published.

Running 1001 time series is a formidable and time-consuming task. It was decided, therefore, by the organizer of this competition, to allow some of the participants to run 111 series only. These 111 series were selected through a random systematic sample. The series in this sample were every ninth entry starting with series 4 (a randomly selected starting point): 4, 13, 22, 31, ..., 994. These 111, as well as the remaining 890 series, are different from the 111 series used in the study reported in JRSS (Makridakis and Hibon, 1979). The Box-Jenkins, Lewandowski, and Parzen methodologies utilized the same systematic sample of 111 series, whereas the rest employed all series. The various tables are, therefore, presented in terms of both the 111 series for all methods and the 1001 series for all methods except the three above-mentioned methods.

### SUMMARY MEASURES OF OVERALL ACCURACY

What are the most appropriate accuracy measures to describe the results of this competition? The answer obviously depends upon the situation involved and the person making the choice. It was decided, therefore, to utilize many important accuracy measures. Interestingly enough, the performance of the various methods differs—sometimes considerably—depending upon the accuracy measure (criterion) being used.

Five summary accuracy measures are reported in this paper: Mean Average Percentage Error (MAPE), Mean Square Error (MSE), Average Ranking (AR), Medians of absolute percentage errors (Md), and Percentage Better (PB).

Table 2(a) shows the MAPE for each method for all 1001 series, whereas Table 2(b) shows the MAPE for the 111 series.

Tables 3(a) and 3(b) show the MSE for each method for the 1001 and 111 series respectively.

It should be noted that a few series whose MAPE were more than 1000% were excluded (this is why not all methods in the various tables of MAPE and MSE have the same  $n(\max)$  value—see Tables 2(a), 2(b), 3(s) and 3(b)).

Tables 4(a) and 4(b) show the AR for each method for all and the 111 series.

Tables 5(a) and 5(b) show the Md for each method for all the 111 series.

Finally, Tables 6(a), 6(b), 7(a), 7(b), 8, 9(a), and 9(b) show the percentage of times that methods Naive 1, Naive 2, Box-Jenkins and Winters' exponential smoothing are better than the other methods (these four methods were chosen because the same results were reported in the JRSS paper).

The accuracy measures reported in Tables 2 to 9 are overall averages. A breakdown of most of these measures also exists for each of the major categories (and often subcategories) shown in Table 1. Unfortunately, space restrictions make it impossible to report them in this paper. Findings concerning some subcategories will be given below. Some general conclusions will be provided in a later section of this paper. It is believed, however, that the best way to understand the results is to consult the various tables carefully.



Table 3(a). Average MSE: all data (1001)

| METHODS      | MODEL<br>FITTING | 1        | 2        | 4        | 6        | 8        | 12       | 18       | 1-4      | 1-12     | 1-18     | n(max) |
|--------------|------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|--------|
| NAIVE 1      |                  | 1206E+11 | 9317E+11 | 1476E+12 | 9360E+12 | 3051E+09 | 1574E+10 | 1576E+10 | 1904E+12 | 2031E+12 | 1491E+12 | 1001   |
| Mov.Averag   |                  | 1247E+11 | 2701E+12 | 3245E+12 | 1151E+12 | 2032E+09 | 1345E+10 | 1372E+10 | 3504E+12 | 2398E+12 | 2199E+12 | 1001   |
| Single EXP   |                  | 1267E+11 | 8553E+11 | 1387E+12 | 1984E+12 | 1251E+09 | 9177E+09 | 9293E+09 | 1804E+12 | 1355E+12 | 1433E+12 | 998    |
| ARR EXP      |                  | 1911E+11 | 8594E+11 | 1126E+12 | 1959E+12 | 1156E+09 | 8887E+09 | 8289E+09 | 1756E+12 | 1928E+12 | 1414E+12 | 1001   |
| Holt EXP     |                  | 1178E+11 | 5117E+11 | 7670E+11 | 9468E+11 | 5395E+12 | 3248E+09 | 2205E+10 | 9991E+11 | 1094E+12 | 8067E+11 | 998    |
| Brown EXP    |                  | 1384E+11 | 4044E+11 | 6033E+11 | 9878E+11 | 4698E+12 | 3134E+09 | 1969E+10 | 8645E+11 | 9755E+11 | 7203E+11 | 1001   |
| Qued.EXP     |                  | 1431E+11 | 3242E+11 | 1365E+12 | 3829E+12 | 1255E+10 | 2980E+10 | 2514E+10 | 8645E+11 | 9237E+11 | 1137E+12 | 997    |
| Regression   |                  | 2168E+11 | 7960E+11 | 1249E+12 | 6928E+12 | 1958E+09 | 1713E+10 | 2045E+10 | 1535E+12 | 1540E+12 | 1137E+12 | 1000   |
| NAIVE2       |                  | 1338E+11 | 9316E+11 | 1476E+12 | 9360E+12 | 3051E+09 | 1574E+10 | 1576E+10 | 1904E+12 | 2031E+12 | 1491E+12 | 1001   |
| D Mov.AVTJ   |                  | 1224E+11 | 2701E+12 | 3245E+12 | 1151E+12 | 2032E+09 | 1345E+10 | 1372E+10 | 3504E+12 | 2398E+12 | 2199E+12 | 1001   |
| D Sing EXP   |                  | 1267E+11 | 8553E+11 | 1387E+12 | 1984E+12 | 1251E+09 | 9177E+09 | 9293E+09 | 1804E+12 | 1355E+12 | 1433E+12 | 998    |
| D ARR EXP    |                  | 1909E+11 | 8594E+11 | 1126E+12 | 1959E+12 | 1156E+09 | 8887E+09 | 8289E+09 | 1756E+12 | 1928E+12 | 1414E+12 | 1001   |
| D Holt EXP   |                  | 1175E+11 | 5113E+11 | 7668E+11 | 9468E+11 | 5394E+12 | 3248E+09 | 2205E+10 | 9991E+11 | 1094E+12 | 8067E+11 | 998    |
| D BrownEXP   |                  | 1381E+11 | 4001E+11 | 6032E+11 | 9876E+11 | 4698E+12 | 3134E+09 | 1969E+10 | 8645E+11 | 9755E+11 | 7203E+11 | 1001   |
| D Qued.EXP   |                  | 1430E+11 | 3233E+11 | 1364E+12 | 3825E+12 | 1254E+10 | 2980E+10 | 2514E+10 | 8645E+11 | 9237E+11 | 1137E+12 | 997    |
| D Regress    |                  | 2165E+11 | 7962E+11 | 1249E+12 | 6927E+12 | 1954E+09 | 1652E+10 | 1933E+10 | 1535E+12 | 1540E+12 | 1137E+12 | 1001   |
| WINTERS      |                  | 1175E+11 | 5113E+11 | 7668E+11 | 9467E+11 | 5394E+12 | 3248E+09 | 2205E+10 | 9991E+11 | 1094E+12 | 8067E+11 | 998    |
| Auton. AEP   |                  | 9105E+10 | 4493E+10 | 6687E+11 | 7060E+11 | 4544E+12 | 1141E+10 | 1308E+10 | 8562E+11 | 9134E+11 | 6740E+11 | 1001   |
| Bayesian F   |                  | 2345E+11 | 2173E+11 | 2894E+11 | 1047E+12 | 3266E+09 | 1708E+10 | 2229E+10 | 5946E+11 | 6198E+11 | 4593E+11 | 997    |
| Combinling A |                  | 9389E+10 | 5759E+11 | 8462E+11 | 5901E+12 | 4191E+09 | 1655E+10 | 1908E+10 | 1114E+12 | 1219E+12 | 8967E+11 | 1001   |
| Combinling A |                  | 1027E+11 | 8325E+11 | 1379E+12 | 9110E+12 | 2319E+09 | 8898E+09 | 8693E+09 | 1798E+12 | 1950E+12 | 1430E+12 | 1001   |
| Average      |                  | 1437E+11 | 8287E+11 | 1190E+12 | 1630E+12 | 7027E+12 | 4548E+09 | 1595E+10 | 1571E+12 | 1556E+12 | 1144E+12 |        |

e.g.  
1340E+11=  
.1340 x 10<sup>11</sup>

Table 3(b). Average MSE: all data (111)

| METHODS      | MODEL<br>FITTING | 1        | 2        | 4        | 6        | 8        | 12       | 18       | 1-4      | 1-12     | 1-18     | n(max) |
|--------------|------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|--------|
| NAIVE 1      |                  | 4502E+08 | 3049E+08 | 7775E+08 | 4657E+09 | 1891E+08 | 4320E+08 | 2742E+08 | 2044E+09 | 2164E+09 | 1736E+09 | 111    |
| Mov.Averag   |                  | 4792E+08 | 2297E+09 | 1976E+09 | 5622E+09 | 5409E+09 | 1116E+08 | 4710E+08 | 2492E+08 | 3377E+09 | 2810E+09 | 111    |
| Single EXP   |                  | 3921E+08 | 4161E+08 | 8697E+08 | 5217E+09 | 4984E+09 | 1158E+08 | 5808E+08 | 2284E+08 | 3377E+09 | 2325E+09 | 111    |
| ARR EXP      |                  | 5109E+08 | 5570E+08 | 2011E+09 | 7274E+09 | 6916E+09 | 1306E+08 | 4510E+08 | 3026E+08 | 3316E+09 | 2891E+09 | 111    |
| Holt EXP     |                  | 3836E+08 | 2718E+08 | 3781E+08 | 2193E+09 | 1000E+09 | 1389E+08 | 7728E+08 | 2205E+08 | 1011E+09 | 1062E+09 | 111    |
| Brown EXP    |                  | 3836E+08 | 2718E+08 | 3781E+08 | 2193E+09 | 1000E+09 | 1389E+08 | 7728E+08 | 2205E+08 | 1011E+09 | 1062E+09 | 111    |
| Qued.EXP     |                  | 4191E+08 | 2635E+08 | 4374E+08 | 2190E+09 | 1099E+09 | 5749E+08 | 9348E+08 | 1293E+09 | 1215E+09 | 1324E+09 | 111    |
| Regression   |                  | 3253E+08 | 3345E+08 | 5241E+08 | 2294E+09 | 1641E+09 | 1403E+08 | 8243E+08 | 1137E+09 | 1129E+09 | 9505E+08 | 110    |
| NAIVE2       |                  | 2594E+08 | 2707E+08 | 6000E+08 | 4559E+09 | 4489E+09 | 1259E+08 | 4820E+08 | 1936E+09 | 1996E+09 | 1637E+09 | 111    |
| D Mov.AVTJ   |                  | 2176E+08 | 2316E+08 | 1857E+08 | 5691E+09 | 5324E+09 | 8741E+08 | 6792E+08 | 1171E+09 | 3360E+09 | 3051E+09 | 111    |
| D Sing EXP   |                  | 2186E+08 | 3890E+08 | 5117E+08 | 7564E+08 | 4810E+09 | 1625E+08 | 3350E+08 | 5251E+08 | 2258E+09 | 2155E+09 | 111    |
| D ARR EXP    |                  | 2849E+08 | 5199E+08 | 1793E+08 | 2525E+09 | 6550E+09 | 1090E+08 | 2428E+08 | 5621E+08 | 3209E+09 | 2776E+09 | 111    |
| D Holt EXP   |                  | 1501E+08 | 2301E+08 | 2525E+08 | 2140E+09 | 9188E+08 | 2535E+08 | 5790E+08 | 8529E+08 | 9464E+08 | 9053E+08 | 111    |
| D BrownEXP   |                  | 1693E+08 | 2619E+08 | 3161E+08 | 2360E+09 | 1039E+09 | 4356E+08 | 8036E+08 | 1143E+09 | 1055E+09 | 1073E+09 | 111    |
| D Qued.EXP   |                  | 1783E+08 | 2415E+08 | 2771E+08 | 2123E+09 | 9759E+08 | 7443E+08 | 9877E+08 | 1551E+09 | 947E+08  | 1085E+09 | 111    |
| D Regress    |                  | 1456E+08 | 2640E+08 | 3315E+08 | 2238E+09 | 1455E+09 | 1241E+08 | 4971E+08 | 6623E+08 | 1033E+09 | 9149E+08 | 110    |
| WINTERS      |                  | 1567E+08 | 2159E+08 | 2538E+08 | 7228E+08 | 7347E+08 | 3242E+08 | 8271E+08 | 1004E+09 | 9443E+08 | 8065E+08 | 111    |
| Auton. AEP   |                  | 2377E+08 | 6811E+08 | 4696E+08 | 4011E+09 | 1999E+09 | 2814E+08 | 6256E+08 | 1469E+08 | 1938E+09 | 1763E+09 | 111    |
| Bayesian F   |                  | 6023E+08 | 2641E+08 | 2549E+08 | 1378E+08 | 9407E+08 | 2266E+08 | 3494E+08 | 5723E+08 | 6650E+08 | 6401E+08 | 111    |
| Combinling A |                  | 1894E+08 | 3431E+08 | 5011E+08 | 3501E+09 | 2079E+09 | 2286E+08 | 4810E+08 | 7083E+08 | 1561E+09 | 1409E+09 | 111    |
| Combinling A |                  | 7057E+08 | 3107E+08 | 4512E+08 | 3403E+09 | 1445E+09 | 1409E+08 | 5625E+08 | 1504E+09 | 1504E+09 | 1203E+09 | 111    |
| Box-Jenkins  |                  | N.A      | 5293E+08 | 3422E+08 | 2491E+09 | 1325E+09 | 3433E+08 | 7849E+08 | 1172E+09 | 1233E+09 | 1506E+09 | 111    |
| Levando-ski  |                  | 4079E+08 | 2015E+08 | 1771E+08 | 8245E+08 | 9147E+08 | 1941E+08 | 4196E+08 | 5307E+08 | 7456E+08 | 7621E+08 | 111    |
| Parzen       |                  | 2231E+08 | 7054E+08 | 1522E+08 | 1105E+09 | 9625E+08 | 2806E+08 | 6592E+08 | 1136E+09 | 9494E+08 | 9351E+08 | 111    |
| Average      |                  | 2850E+08 | 5199E+08 | 6793E+08 | 3416E+09 | 2626E+09 | 2737E+08 | 5814E+08 | 6985E+08 | 1653E+09 | 1345E+09 |        |

e.g.  
4510E+08=  
.4510 x 10<sup>8</sup>





Table 5(a). Median APE: all data (1001)

| METHODS     | FORECASTING HORIZONS |     |     |     | CUMULATIVE FORECASTING HORIZONS |      |      |      |      |      |      |
|-------------|----------------------|-----|-----|-----|---------------------------------|------|------|------|------|------|------|
|             | MODEL FITTING        | 1   | 2   | 3   | 4                               | 5    | 6    | 8    | 12   | 15   | 18   |
| NAIVE 1     | 5.9                  | 8.0 | 8.8 | 9.0 | 10.7                            | 12.0 | 13.3 | 12.3 | 10.4 | 13.9 | 15.7 |
| MUV-AVEHAG  | 5.9                  | 8.3 | 8.9 | 9.0 | 10.7                            | 11.7 | 13.0 | 11.6 | 10.4 | 13.9 | 15.7 |
| SINGLE EXP  | 5.9                  | 8.0 | 8.2 | 8.5 | 10.2                            | 11.2 | 12.5 | 11.1 | 12.2 | 13.4 | 14.9 |
| AR EXP      | 6.5                  | 8.7 | 9.7 | 8.5 | 10.2                            | 11.2 | 12.7 | 11.2 | 12.2 | 13.4 | 14.9 |
| MOLT EXP    | 5.3                  | 8.7 | 7.2 | 7.1 | 8.9                             | 9.5  | 11.4 | 11.2 | 11.3 | 13.2 | 13.1 |
| BRO-N EXP   | 5.5                  | 8.3 | 6.9 | 7.6 | 8.9                             | 10.0 | 11.8 | 12.0 | 14.6 | 15.3 | 13.1 |
| QUAD EXP    | 7.6                  | 9.0 | 7.8 | 8.9 | 9.8                             | 12.0 | 14.3 | 16.2 | 11.7 | 12.6 | 12.6 |
| REGRESSION  | 3.6                  | 4.8 | 6.6 | 6.9 | 8.8                             | 9.9  | 10.8 | 10.4 | 10.4 | 13.1 | 14.1 |
| NAIVE2      | 3.6                  | 5.8 | 7.9 | 9.5 | 11.6                            | 12.3 | 13.8 | 11.5 | 11.5 | 14.1 | 14.3 |
| D MUV-AVRG  | 3.6                  | 4.7 | 6.2 | 6.8 | 8.2                             | 9.0  | 10.5 | 10.0 | 10.0 | 12.6 | 12.3 |
| D SING EXP  | 4.1                  | 5.3 | 7.3 | 7.1 | 8.6                             | 9.6  | 10.1 | 9.2  | 9.9  | 12.2 | 13.6 |
| D MOLT EXP  | 3.3                  | 4.5 | 5.3 | 5.6 | 7.3                             | 8.5  | 9.6  | 10.0 | 11.1 | 12.7 | 15.9 |
| D BRO-N EXP | 3.4                  | 4.4 | 5.1 | 6.2 | 7.3                             | 8.5  | 10.0 | 11.7 | 13.9 | 15.6 | 20.4 |
| D QUAD EXP  | 3.4                  | 4.5 | 5.1 | 6.3 | 7.9                             | 10.0 | 11.7 | 13.9 | 15.6 | 20.4 | 11.9 |
| D REGRESS   | 5.6                  | 7.8 | 7.9 | 9.1 | 9.1                             | 9.9  | 9.6  | 10.7 | 10.1 | 11.4 | 12.2 |
| WINTERS     | 3.7                  | 4.4 | 5.4 | 5.5 | 6.6                             | 8.5  | 9.7  | 9.6  | 10.5 | 12.4 | 13.1 |
| AUTOM. AEP  | 4.0                  | 4.7 | 5.8 | 6.6 | 7.2                             | 8.5  | 10.0 | 10.2 | 10.1 | 12.5 | 13.1 |
| RAYESIAN F  | 5.0                  | 5.1 | 5.5 | 6.4 | 7.8                             | 8.5  | 10.0 | 10.2 | 10.1 | 12.5 | 13.1 |
| COMBINING A | 3.3                  | 4.2 | 5.4 | 5.9 | 7.1                             | 8.0  | 9.7  | 9.3  | 9.5  | 11.9 | 12.6 |
| COMBINING B | 3.4                  | 4.6 | 6.0 | 6.6 | 7.7                             | 8.8  | 10.1 | 10.1 | 10.4 | 12.3 | 12.8 |

Table 5(b). Median APE: all data (111)

| METHODS     | FORECASTING HORIZONS |     |      |      | CUMULATIVE FORECASTING HORIZONS |      |      |      |      |      |      |
|-------------|----------------------|-----|------|------|---------------------------------|------|------|------|------|------|------|
|             | MODEL FITTING        | 1   | 2    | 3    | 4                               | 5    | 6    | 8    | 12   | 15   | 18   |
| NAIVE 1     | 6.1                  | 5.3 | 9.4  | 10.3 | 12.2                            | 13.1 | 14.3 | 14.7 | 7.9  | 18.1 | 18.3 |
| MUV-AVEHAG  | 5.7                  | 5.6 | 8.9  | 10.0 | 12.2                            | 12.0 | 13.3 | 10.7 | 11.7 | 17.0 | 17.5 |
| SINGLE EXP  | 5.9                  | 5.2 | 7.4  | 9.6  | 11.2                            | 13.3 | 13.5 | 11.3 | 11.7 | 17.0 | 19.5 |
| AR EXP      | 5.5                  | 4.3 | 9.3  | 9.4  | 11.7                            | 12.5 | 13.8 | 12.8 | 10.9 | 18.9 | 16.2 |
| MOLT EXP    | 5.7                  | 4.6 | 7.7  | 4.1  | 10.3                            | 11.7 | 13.8 | 12.8 | 10.9 | 18.9 | 16.2 |
| BRO-N EXP   | 5.7                  | 5.5 | 7.7  | 9.7  | 11.0                            | 12.6 | 15.0 | 13.3 | 12.3 | 24.9 | 16.1 |
| QUAD EXP    | 7.9                  | 9.9 | 10.4 | 12.4 | 15.0                            | 15.0 | 15.0 | 10.3 | 10.3 | 12.9 | 15.7 |
| REGRESSION  | 3.6                  | 4.0 | 5.8  | 17.3 | 10.1                            | 18.7 | 18.7 | 10.3 | 10.3 | 12.9 | 15.7 |
| NAIVE2      | 3.6                  | 5.2 | 8.0  | 10.6 | 13.5                            | 15.1 | 16.3 | 10.3 | 10.3 | 12.9 | 15.7 |
| D MUV-AVRG  | 3.2                  | 4.7 | 5.9  | 6.9  | 8.6                             | 9.6  | 11.1 | 9.9  | 10.0 | 12.9 | 15.6 |
| D SING EXP  | 3.9                  | 4.7 | 7.2  | 7.3  | 9.6                             | 9.6  | 11.1 | 10.2 | 10.0 | 12.9 | 15.6 |
| D MOLT EXP  | 3.1                  | 3.4 | 5.0  | 6.4  | 8.1                             | 7.4  | 9.0  | 10.6 | 9.7  | 12.7 | 15.0 |
| D BRO-N EXP | 3.1                  | 3.3 | 4.3  | 6.9  | 8.0                             | 8.6  | 9.2  | 10.6 | 9.7  | 12.7 | 15.0 |
| D QUAD EXP  | 3.1                  | 3.7 | 5.1  | 7.9  | 7.2                             | 8.9  | 9.0  | 13.3 | 10.6 | 16.2 | 20.1 |
| D REGRESS   | 5.4                  | 6.7 | 7.7  | 9.5  | 9.4                             | 10.7 | 13.1 | 12.1 | 10.6 | 16.2 | 13.7 |
| WINTERS     | 3.5                  | 4.1 | 5.2  | 5.9  | 7.0                             | 6.5  | 7.6  | 10.6 | 9.2  | 13.7 | 15.2 |
| AUTOM. AEP  | 4.1                  | 4.6 | 6.2  | 7.0  | 8.7                             | 8.3  | 7.6  | 10.6 | 9.2  | 13.7 | 15.2 |
| RAYESIAN F  | 5.0                  | 5.0 | 5.3  | 5.9  | 7.0                             | 7.3  | 8.7  | 12.1 | 8.6  | 12.9 | 14.8 |
| COMBINING A | 3.2                  | 2.9 | 5.0  | 6.0  | 7.7                             | 6.9  | 8.7  | 9.9  | 8.6  | 12.9 | 14.8 |
| COMBINING B | 3.1                  | 3.3 | 5.8  | 6.1  | 7.8                             | 7.1  | 7.5  | 9.3  | 8.6  | 12.9 | 14.8 |
| BOX-JENKINS | 0.0                  | 5.3 | 5.0  | 5.2  | 6.6                             | 6.0  | 8.9  | 9.3  | 8.6  | 12.9 | 14.8 |
| LEANDRO-SKI | 3.7                  | 5.4 | 6.1  | 5.3  | 7.5                             | 7.9  | 7.9  | 9.3  | 8.6  | 12.9 | 14.8 |
| PARZEN      | 3.7                  | 4.8 | 6.1  | 6.3  | 7.6                             | 8.5  | 9.0  | 9.1  | 8.6  | 11.5 | 11.6 |





Table 8. Percentage of time that the Box-Jenkins method is better than other methods ( $n = 111$ )

| METHODS     | MODEL FITTING | Forecasting Horizons |      |      |      |      |      |      |      |      |      |      | AVERAGE OF ALL FORECASTS |      |      |      |      |      |      |       |
|-------------|---------------|----------------------|------|------|------|------|------|------|------|------|------|------|--------------------------|------|------|------|------|------|------|-------|
|             |               | 1                    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 11   | 12                       | 13   | 14   | 15   | 16   | 17   | 18   |       |
| NATIVE 1    | N.A           | 58.6                 | 68.5 | 68.5 | 60.4 | 70.3 | 64.0 | 62.6 | 62.6 | 63.2 | 48.5 | 44.1 | 42.6                     | 41.2 | 57.4 | 63.2 | 55.9 | 55.9 | 55.9 | 59.29 |
| Mov.Average | N.A           | 56.9                 | 61.0 | 63.1 | 62.2 | 72.1 | 67.6 | 62.6 | 65.9 | 58.8 | 51.5 | 47.1 | 52.9                     | 41.2 | 58.8 | 60.3 | 52.9 | 51.5 | 54.4 | 59.23 |
| Single Exp  | N.A           | 55.0                 | 59.5 | 64.9 | 64.0 | 64.7 | 66.7 | 60.4 | 62.6 | 55.9 | 51.5 | 50.0 | 51.5                     | 41.2 | 57.4 | 61.8 | 54.4 | 57.4 | 54.4 | 58.51 |
| ARR EXP     | N.A           | 57.7                 | 63.1 | 66.7 | 64.0 | 65.9 | 69.4 | 59.3 | 62.6 | 58.8 | 50.0 | 45.6 | 51.5                     | 47.1 | 51.5 | 60.3 | 60.3 | 57.4 | 51.5 | 59.10 |
| Molt Exp    | N.A           | 47.7                 | 55.9 | 54.1 | 52.3 | 56.8 | 53.2 | 57.1 | 58.2 | 59.8 | 51.5 | 58.8 | 51.5                     | 47.1 | 55.9 | 61.8 | 54.4 | 54.4 | 45.6 | 54.12 |
| Brown Exp   | N.A           | 50.5                 | 59.9 | 59.5 | 53.2 | 60.4 | 53.2 | 59.3 | 58.2 | 60.3 | 50.0 | 55.9 | 54.4                     | 48.5 | 50.0 | 58.8 | 51.5 | 54.4 | 51.5 | 55.27 |
| Quard Exp   | N.A           | 52.3                 | 63.1 | 65.8 | 55.9 | 66.7 | 62.2 | 68.1 | 68.1 | 61.8 | 58.8 | 63.2 | 66.2                     | 60.3 | 72.1 | 70.6 | 69.1 | 60.3 | 69.1 | 63.68 |
| Regression  | N.A           | 64.0                 | 68.5 | 72.1 | 65.8 | 67.6 | 66.7 | 63.7 | 67.0 | 61.8 | 58.8 | 66.2 | 61.8                     | 55.9 | 61.8 | 61.8 | 51.5 | 54.4 | 50.0 | 63.15 |
| MATVE2      | N.A           | 46.8                 | 56.8 | 61.3 | 59.5 | 61.3 | 55.0 | 54.9 | 53.8 | 58.8 | 48.5 | 44.1 | 42.6                     | 42.6 | 48.5 | 60.3 | 50.0 | 52.9 | 48.5 | 53.34 |
| D Mov.Avr 3 | N.A           | 55.9                 | 61.3 | 69.4 | 65.8 | 71.2 | 69.4 | 65.9 | 65.9 | 60.3 | 51.5 | 58.8 | 54.4                     | 47.1 | 52.9 | 55.9 | 60.3 | 64.7 | 54.4 | 61.32 |
| D Sing EXP  | N.A           | 49.5                 | 52.3 | 62.2 | 56.8 | 60.4 | 59.5 | 50.5 | 53.8 | 55.9 | 41.2 | 45.6 | 48.5                     | 41.2 | 44.1 | 55.9 | 50.0 | 52.9 | 51.5 | 52.29 |
| D ARR EXP   | N.A           | 49.5                 | 57.7 | 62.2 | 59.5 | 64.9 | 60.4 | 49.5 | 58.2 | 52.9 | 42.6 | 41.2 | 47.1                     | 41.2 | 45.6 | 57.4 | 48.5 | 54.4 | 55.9 | 53.80 |
| D Holt EXP  | N.A           | 39.6                 | 45.5 | 45.0 | 47.7 | 45.9 | 45.0 | 53.8 | 47.3 | 55.9 | 51.5 | 50.0 | 48.5                     | 45.6 | 38.2 | 61.8 | 52.9 | 54.4 | 52.9 | 48.33 |
| D BrownEXP  | N.A           | 44.1                 | 43.7 | 49.5 | 46.8 | 47.7 | 52.3 | 52.7 | 56.0 | 47.1 | 52.9 | 54.4 | 54.4                     | 51.5 | 47.1 | 55.9 | 54.4 | 54.4 | 48.5 | 50.29 |
| D Quard EXP | N.A           | 43.2                 | 49.5 | 55.9 | 48.6 | 56.8 | 54.1 | 57.1 | 61.5 | 61.8 | 57.4 | 55.9 | 58.8                     | 58.8 | 54.4 | 66.2 | 64.7 | 61.8 | 60.3 | 56.15 |
| D Regress   | N.A           | 59.5                 | 55.0 | 65.8 | 64.0 | 56.8 | 61.3 | 58.2 | 57.1 | 51.5 | 52.9 | 55.9 | 51.5                     | 50.0 | 47.1 | 60.3 | 47.1 | 52.9 | 52.9 | 56.41 |
| WINTERS     | N.A           | 50.5                 | 44.1 | 48.6 | 45.0 | 45.9 | 46.8 | 51.6 | 46.2 | 50.0 | 39.7 | 50.0 | 48.5                     | 45.6 | 36.8 | 50.0 | 52.9 | 50.0 | 41.2 | 46.92 |
| Autom. AEP  | N.A           | 45.9                 | 52.3 | 54.1 | 50.5 | 55.0 | 52.3 | 50.3 | 54.9 | 58.8 | 48.5 | 48.5 | 45.6                     | 44.1 | 51.5 | 54.4 | 47.1 | 50.0 | 61.0 | 52.03 |
| Bavesian F  | N.A           | 51.4                 | 54.1 | 56.8 | 54.1 | 51.4 | 54.1 | 54.9 | 60.4 | 52.9 | 51.5 | 45.6 | 47.1                     | 39.7 | 41.2 | 48.5 | 50.0 | 42.6 | 45.6 | 50.92 |
| Combining F | N.A           | 39.6                 | 45.9 | 48.6 | 48.6 | 54.1 | 53.2 | 53.8 | 49.5 | 52.9 | 39.7 | 39.7 | 45.6                     | 39.7 | 38.2 | 55.9 | 48.5 | 44.1 | 45.6 | 47.25 |
| Combining R | N.A           | 41.4                 | 49.5 | 50.5 | 55.9 | 49.5 | 52.3 | 49.5 | 51.6 | 55.9 | 36.8 | 42.6 | 48.5                     | 44.1 | 44.1 | 55.9 | 50.0 | 54.4 | 51.5 | 48.28 |
| Lewandowsk1 | N.A           | 56.8                 | 55.9 | 61.3 | 55.0 | 52.3 | 50.5 | 39.6 | 45.1 | 52.9 | 44.1 | 39.7 | 45.6                     | 47.1 | 36.8 | 47.1 | 45.6 | 36.8 | 38.2 | 48.43 |
| Parzen      | N.A           | 49.5                 | 45.0 | 49.5 | 45.9 | 54.1 | 50.5 | 50.5 | 48.4 | 52.9 | 48.5 | 49.7 | 42.6                     | 57.4 | 48.5 | 45.6 | 39.7 | 45.6 | 50.0 | 48.23 |
| Average     | N.A           | 50.5                 | 55.2 | 58.9 | 55.7 | 58.8 | 57.3 | 56.1 | 57.2 | 56.5 | 49.0 | 49.7 | 50.5                     | 46.9 | 49.6 | 57.8 | 52.7 | 52.9 | 51.0 | 54.23 |



## EFFECTS OF SAMPLING

Can the results of this study be generalized? Surprisingly, not much is known about the sampling distribution of actual post-sample forecasting errors. Furthermore, not much is known about the relative desirability of different accuracy measures for the purpose of comparing various forecasting methods.

The five accuracy measures reported in this study (i.e. MAPE, MSE, AR, Md and PB) are not exhaustive. Average Percentage Errors (APE), Mean Absolute Deviations (MAD), Mean Root Square Errors (MRSE), and other accuracy measures could have been used (APE and MAD have been computed but are not reported in this paper).

Having to report the accuracy measures for both the 1001 and 111 series is a disadvantage because it increases the length of the paper and the time and effort required to read it. The advantage, however, is that the reader can examine how each of the five accuracy measures differs between the 111 and all 1001 series for the 21 methods that are reported on both the 111 and 1001 series. Although the 111 series is only a part of the 1001, much can be learned by looking at the (a) and (b) parts of Tables 2 to 9 and seeing how the various accuracy measures vary among the (a) and (b) parts.

In general, the MSE fluctuates much more than the other measures, whereas Md, PB and AR fluctuate the least with MAPE somewhere in between. For instance, the overall average MSE of the Automatic AEP method is one of the best for all 1001 series and one of the worst for the 111 series. On the other hand, the other four measures are more consistent between the (a) and (b) parts of the tables.

In order to obtain a more precise idea of sampling variations, Table 10 shows the behaviour of five measures for nine systematic samples from the 1001 series for a single method, chosen arbitrarily: Holt-Winters exponential smoothing. It is not difficult to see that the variations in the results from the nine different samples are relatively smaller for MAPE than for MSE while the average rankings and the percentage better measures seem to fluctuate the least.

Would the results of the systematic samples for the Holt-Winters method vary more if other data were used? To deal with this type of question, Table 11 compares the percentage of times the Box-Jenkins methodology was better than other methods used, both in the present study and in that reported in JRSS. (The entries for Table 11 have been taken from Table 7 of the present study and Table 6 on p. 108 of the JRSS paper.) The results do vary, as can be expected, but for most methods they are similar, in particular for the overall average.

## SOME GENERAL OBSERVATIONS

The performance of various methods differs considerably sometimes, depending upon the accuracy measure (criterion) being used. Parzen and Holt-Winters are two methods which exhibit a higher degree of consistency among most of the five accuracy measures than the remaining methods.

Differences among methods were influenced by differences in the type of series used and the length of the forecasting horizon. These differences are discussed next mainly within the subset of the 111 series.

### Effects of the type of series

The relative forecasting accuracy of the various methods was affected significantly by (a) the yearly, quarterly or monthly nature of data; (b) the micro, macro classification; and (c) whether the data

Table 10.

| METHODS  | AVERAGE MAPE : ALL DATA (n=111)  |           |           |           |           |           |           |           |           |                                 |                          |      |      |      |      |      |        |      |
|----------|----------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|---------------------------------|--------------------------|------|------|------|------|------|--------|------|
|          | Forecasting Horizons             |           |           |           |           |           |           |           |           | Average of Forecasting Horizons |                          |      |      |      |      |      |        |      |
|          | 1                                | 2         | 3         | 4         | 5         | 6         | 8         | 12        | 15        | 18                              | 1-4                      | 1-6  | 1-8  | 1-12 | 1-15 | 1-18 | n(max) |      |
| FITTING  |                                  |           |           |           |           |           |           |           |           |                                 |                          |      |      |      |      |      |        |      |
| Winter 1 | 9.3                              | 9.0       | 11.2      | 11.6      | 13.7      | 17.8      | 21.2      | 24.0      | 12.4      | 16.9                            | 20.7                     | 11.9 | 14.8 | 16.5 | 16.0 | 16.1 | 16.5   | 110  |
| Winter 2 | 9.3                              | 9.2       | 10.5      | 13.4      | 15.5      | 17.5      | 18.7      | 23.3      | 15.9      | 13.4                            | 34.5                     | 12.1 | 14.1 | 16.3 | 16.4 | 17.8 | 19.5   | 111  |
| Winter 3 | 8.4                              | 3.8       | 10.5      | 14.4      | 12.0      | 15.9      | 24.7      | 21.3      | 23.5      | 27.4                            | 34.6                     | 11.4 | 14.3 | 15.6 | 16.7 | 17.5 | 18.7   | 111  |
| Winter 4 | 17.2                             | 4.6       | 11.4      | 12.6      | 16.5      | 13.4      | 22.3      | 27.0      | 24.5      | 28.0                            | 35.9                     | 12.4 | 15.0 | 17.3 | 20.9 | 22.3 | 23.7   | 111  |
| Winter 5 | 9.9                              | 9.3       | 11.4      | 16.4      | 17.0      | 22.7      | 22.0      | 21.1      | 17.1      | 22.2                            | 39.4                     | 13.6 | 16.5 | 17.2 | 16.9 | 17.6 | 19.9   | 110  |
| Winter 6 | 9.4                              | 8.0       | 10.0      | 12.7      | 16.6      | 18.8      | 19.6      | 26.5      | 28.8      | 37.6                            | 62.4                     | 11.8 | 14.3 | 16.4 | 19.1 | 21.8 | 25.4   | 111  |
| Winter 7 | 7.5                              | 6.9       | 7.8       | 10.7      | 12.8      | 14.4      | 16.0      | 16.3      | 21.6      | 33.9                            | 67.1                     | 9.5  | 11.4 | 12.4 | 14.6 | 16.8 | 21.1   | 110  |
| Winter 8 | 8.9                              | 7.4       | 9.8       | 12.3      | 12.0      | 25.0      | 25.4      | 37.6      | 32.5      | 54.3                            | 86.4                     | 10.4 | 15.3 | 19.1 | 22.1 | 24.9 | 30.7   | 111  |
| Winter 9 | 9.3                              | 10.3      | 13.5      | 14.8      | 17.1      | 19.8      | 21.0      | 18.8      | 27.5      | 38.6                            | 41.3                     | 13.0 | 16.1 | 16.7 | 19.0 | 21.8 | 24.0   | 111  |
| Average  | 9.3                              | 8.7       | 10.9      | 13.2      | 14.8      | 18.9      | 21.4      | 24.0      | 22.7      | 32.5                            | 46.9                     | 11.9 | 14.7 | 16.4 | 18.0 | 19.6 | 22.2   |      |
| -----    |                                  |           |           |           |           |           |           |           |           |                                 |                          |      |      |      |      |      |        |      |
| METHODS  | AVERAGE MAPE : ALL DATA (n=111)  |           |           |           |           |           |           |           |           |                                 |                          |      |      |      |      |      |        |      |
|          | Forecasting Horizons             |           |           |           |           |           |           |           |           | Average of Forecasting Horizons |                          |      |      |      |      |      |        |      |
|          | 1                                | 2         | 3         | 4         | 5         | 6         | 8         | 12        | 15        | 18                              | 1-4                      | 1-6  | 1-8  | 1-12 | 1-15 | 1-18 | n(max) |      |
| FITTING  |                                  |           |           |           |           |           |           |           |           |                                 |                          |      |      |      |      |      |        |      |
| Winter 1 | .6965E+08                        | .1308E+09 | .1345E+09 | .1328E+10 | .4857E+10 | .6196E+08 | .7221E+07 | .1790E+08 | .4660E+09 | .7891E+09                       | .5905E+09                | .110 |      |      |      |      |        |      |
| Winter 2 | .1567E+08                        | .2158E+08 | .2538E+08 | .9347E+08 | .2842E+08 | .8271E+08 | .1004E+09 | .8448E+08 | .9448E+08 | .9393E+08                       | .111                     |      |      |      |      |      |        |      |
| Winter 3 | .2266E+11                        | .4191E+12 | .6340E+12 | .6227E+12 | .4665E+13 | .1536E+09 | .3300E+09 | .1215E+09 | .8086E+12 | .6095E+12                       | .6667E+12                | .111 |      |      |      |      |        |      |
| Winter 4 | .1385E+11                        | .1620E+11 | .3461E+11 | .2198E+09 | .2994E+11 | .2972E+09 | .6083E+08 | .1008E+08 | .1329E+11 | .8644E+10                       | .6340E+10                | .111 |      |      |      |      |        |      |
| Winter 5 | .1308E+10                        | .1242E+10 | .1815E+10 | .5398E+10 | .1016E+11 | .1496E+09 | .7458E+07 | .7050E+08 | .4367E+10 | .3416E+10                       | .2312E+10                | .111 |      |      |      |      |        |      |
| Winter 6 | .3906E+11                        | .3016E+10 | .2187E+10 | .1961E+12 | .1204E+11 | .8029E+09 | .1250E+10 | .2285E+10 | .5337E+11 | .3818E+11                       | .2847E+11                | .111 |      |      |      |      |        |      |
| Winter 7 | .1908E+11                        | .1215E+11 | .1158E+11 | .1041E+11 | .7350E+11 | .4576E+08 | .6269E+07 | .3138E+08 | .9038E+10 | .1226E+11                       | .8980E+10                | .110 |      |      |      |      |        |      |
| Winter 8 | .6896E+10                        | .9812E+10 | .4911E+10 | .1212E+11 | .5011E+11 | .1761E+08 | .1381E+08 | .7139E+07 | .7953E+10 | .1066E+11                       | .7816E+10                | .111 |      |      |      |      |        |      |
| Winter 9 | .1047E+08                        | .2638E+08 | .7233E+08 | .9885E+08 | .2921E+08 | .8246E+08 | .1073E+08 | .9330E+08 | .3953E+08 | .3409E+08                       | .4136E+08                | .111 |      |      |      |      |        |      |
| Average  | .1176E+11                        | .5108E+11 | .7560E+11 | .9429E+11 | .5384E+12 | .4853E+09 | .1944E+09 | .3042E+09 | .9969E+11 | .1093E+12                       | .8017E+11                |      |      |      |      |      |        |      |
| -----    |                                  |           |           |           |           |           |           |           |           |                                 |                          |      |      |      |      |      |        |      |
| METHODS  | AVERAGE RANKING : ALL DATA (111) |           |           |           |           |           |           |           |           |                                 |                          |      |      |      |      |      |        |      |
|          | Forecasting Horizons             |           |           |           |           |           |           |           |           | Average of ALL FORECASTS        |                          |      |      |      |      |      |        |      |
|          | 1                                | 2         | 3         | 4         | 5         | 6         | 8         | 12        | 15        | 18                              | AVERAGE OF ALL FORECASTS |      |      |      |      |      |        |      |
| FITTING  |                                  |           |           |           |           |           |           |           |           |                                 |                          |      |      |      |      |      |        |      |
| Winter 1 | 5.2                              | 5.3       | 5.3       | 5.1       | 5.1       | 5.1       | 5.0       | 5.0       | 4.8       | 4.9                             | 5.2                      | 5.2  | 4.9  | 5.1  | 5.1  | 4.8  | 4.8    | 5.06 |
| Winter 2 | 5.0                              | 5.2       | 5.0       | 4.9       | 5.0       | 5.0       | 4.8       | 5.1       | 5.2       | 4.8                             | 4.8                      | 5.0  | 4.9  | 5.0  | 5.0  | 4.8  | 5.1    | 4.99 |
| Winter 3 | 4.6                              | 4.7       | 4.8       | 4.8       | 4.6       | 4.7       | 4.7       | 4.4       | 4.7       | 4.4                             | 4.3                      | 4.1  | 4.4  | 4.2  | 4.3  | 4.3  | 4.6    | 4.55 |
| Winter 4 | 4.9                              | 4.7       | 5.0       | 4.9       | 4.9       | 4.9       | 5.1       | 5.0       | 5.1       | 5.2                             | 5.1                      | 5.0  | 4.8  | 4.8  | 5.3  | 4.9  | 4.9    | 4.95 |
| Winter 5 | 5.2                              | 5.2       | 5.1       | 4.8       | 5.1       | 5.1       | 5.1       | 4.9       | 5.1       | 5.2                             | 4.8                      | 4.8  | 4.9  | 4.8  | 4.7  | 5.0  | 4.7    | 4.98 |
| Winter 6 | 5.5                              | 5.3       | 5.4       | 5.5       | 5.4       | 5.4       | 5.6       | 5.5       | 5.7       | 5.8                             | 6.1                      | 5.9  | 5.7  | 5.8  | 5.9  | 6.1  | 5.9    | 5.67 |
| Winter 7 | 5.2                              | 5.3       | 5.3       | 5.5       | 5.5       | 5.5       | 5.6       | 5.0       | 4.9       | 5.3                             | 5.2                      | 5.0  | 5.2  | 5.2  | 5.2  | 5.1  | 5.1    | 5.23 |
| Winter 8 | 4.7                              | 4.5       | 4.5       | 4.6       | 4.5       | 4.4       | 4.5       | 4.6       | 4.9       | 4.9                             | 4.7                      | 4.9  | 4.7  | 5.0  | 4.8  | 4.7  | 4.8    | 4.70 |
| Winter 9 | 4.7                              | 4.8       | 4.7       | 4.9       | 4.9       | 4.8       | 4.7       | 5.0       | 4.7       | 4.9                             | 5.1                      | 5.0  | 4.9  | 5.1  | 4.8  | 5.0  | 5.1    | 4.88 |
| Average  | 5.0                              | 5.0       | 5.0       | 5.0       | 5.0       | 5.0       | 5.0       | 5.0       | 5.0       | 5.0                             | 5.0                      | 5.0  | 5.0  | 5.0  | 5.0  | 5.0  | 5.0    | 5.00 |

e.g.  
 $.6965E+08 =$   
 $.6965 \times 10^8$





Table 11. Percentage of times that the Box-Jenkins method was better than methods listed for the study reported in JRSS and the present competition

| Forecasting Method              | Forecasting Horizons <sup>1</sup> |      |      |      |      |      |      |      |      |      |      |      |      |      | Simple Average |      |       |       |
|---------------------------------|-----------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|----------------|------|-------|-------|
|                                 | 1                                 |      | 2    |      | 3    |      | 4    |      | 5    |      | 6    |      | 9    |      |                |      | 12    |       |
|                                 | JRSS                              | Now  | JRSS | Now  | JRSS | Now  | JRSS | Now  | JRSS | Now  | JRSS | Now  | JRSS | Now  | JRSS           | Now  | JRSS  | Now   |
| 1. Naive 1                      | 59.1                              | 58.6 | 63.6 | 68.5 | 57.3 | 68.5 | 63.6 | 60.4 | 64.5 | 70.3 | 59.1 | 64.0 | 63.6 | 63.2 | 67.3           | 42.6 | 62.26 | 62.01 |
| 2. Single Mov. Average          | 49.1                              | 64.9 | 56.4 | 66.7 | 53.6 | 71.2 | 60.9 | 68.5 | 64.5 | 66.7 | 61.8 | 64.9 | 65.5 | 50.0 | 66.4           | 54.4 | 59.77 | 63.41 |
| 3. Single Exp. Smoothing        | 53.6                              | 55.0 | 57.3 | 59.5 | 55.5 | 64.9 | 60.0 | 64.0 | 62.7 | 66.7 | 62.7 | 66.7 | 65.5 | 55.9 | 66.4           | 51.5 | 60.46 | 60.52 |
| 4. Adapt.                       | 49.1                              | 57.7 | 56.4 | 63.1 | 56.4 | 66.7 | 60.0 | 64.0 | 62.7 | 65.8 | 64.5 | 69.4 | 62.7 | 58.8 | 67.3           | 51.5 | 59.88 | 62.12 |
| 5. Linear Mov. Average          |                                   | NA   |      | NA   |      | NA   |      | NA   |      | NA   |      | NA   |      | NA   |                |      |       | NA    |
| 6. Brown's                      | 42.7                              | 50.5 | 51.8 | 59.9 | 50.0 | 59.5 | 53.6 | 53.2 | 59.1 | 60.4 | 59.1 | 53.2 | 67.3 | 60.3 | 70.9           | 54.4 | 56.81 | 56.42 |
| 7. Holt's                       | 50.0                              | 47.7 | 55.5 | 55.9 | 50.0 | 54.1 | 54.5 | 52.3 | 62.7 | 56.8 | 61.8 | 53.2 | 62.7 | 58.8 | 66.4           | 51.5 | 57.95 | 53.79 |
| 8. Brown's Quadr.               | 48.2                              | 52.3 | 51.8 | 63.1 | 50.9 | 65.8 | 57.3 | 55.9 | 61.8 | 66.7 | 63.6 | 62.2 | 65.5 | 61.8 | 73.6           | 66.2 | 59.09 | 61.75 |
| 9. Linear Trend                 | 60.9                              | 64.0 | 61.8 | 68.5 | 63.6 | 72.1 | 65.5 | 65.8 | 69.1 | 67.6 | 68.2 | 66.7 | 71.8 | 61.8 | 72.7           | 61.8 | 66.7  | 66.04 |
| 10. Harrison's                  |                                   | NA   |      | NA   |      | NA   |      | NA   |      | NA   |      | NA   |      | NA   |                |      |       | NA    |
| 11. Winters'                    | 47.3                              | 50.5 | 49.1 | 44.1 | 51.8 | 48.6 | 50.0 | 45.0 | 50.0 | 45.9 | 50.9 | 46.8 | 46.4 | 50.0 | 45.5           | 48.5 | 48.88 | 47.43 |
| 12. Adaptive Filt. <sup>2</sup> | 49.1                              | 45.9 | 54.5 | 52.3 | 52.7 | 54.1 | 55.5 | 50.5 | 56.4 | 55.0 | 56.4 | 52.3 | 58.2 | 58.8 | 51.8           | 45.6 | 54.33 | 51.81 |
| 13. Naive 2                     | 53.6                              | 46.8 | 46.4 | 56.8 | 44.5 | 61.3 | 42.7 | 59.5 | 48.2 | 61.3 | 41.8 | 55.0 | 38.2 | 58.8 | 39.1           | 42.6 | 44.31 | 55.26 |
| 14. Single Mov. Average         | 46.4                              | 70.3 | 42.7 | 70.3 | 43.6 | 72.1 | 39.1 | 67.6 | 42.7 | 70.3 | 39.1 | 65.8 | 37.3 | 58.8 | 38.2           | 42.6 | 41.14 | 64.73 |
| 15. Single Exp. Smoothing       | 50.0                              | 45.0 | 42.7 | 52.3 | 42.7 | 62.2 | 38.2 | 56.8 | 43.6 | 60.4 | 39.1 | 59.5 | 41.8 | 55.9 | 40.0           | 48.5 | 42.26 | 55.07 |
| 16. Adaptive Resp.              | 44.5                              | 49.5 | 46.4 | 57.7 | 42.7 | 62.2 | 40.9 | 59.5 | 47.3 | 64.9 | 48.2 | 60.4 | 44.5 | 52.9 | 41.8           | 47.1 | 44.54 | 56.77 |
| 17. Linear Mov. Average         |                                   | NA   |      | NA   |      | NA   |      | NA   |      | NA   |      | NA   |      | NA   |                |      |       | NA    |
| 18. Brown's Linear              | 42.7                              | 44.1 | 48.2 | 43.7 | 44.5 | 49.5 | 44.5 | 46.8 | 49.1 | 47.7 | 47.3 | 52.3 | 49.1 | 47.1 | 46.4           | 54.4 | 46.48 | 48.2  |
| 19. Holt's Linear               | 41.8                              | 39.6 | 43.6 | 45.5 | 42.7 | 45.0 | 40.0 | 47.7 | 46.4 | 45.9 | 44.5 | 45.0 | 46.4 | 55.9 | 45.5           | 48.5 | 43.86 | 46.64 |
| 20. Brown's Quadrat.            | 41.8                              | 43.2 | 44.5 | 49.5 | 44.5 | 55.9 | 45.5 | 48.6 | 47.3 | 56.8 | 47.3 | 54.1 | 50.9 | 61.8 | 59.1           | 58.8 | 47.61 | 53.59 |
| 21. Linear Trend                | 63.6                              | 59.5 | 59.1 | 55.0 | 62.7 | 65.8 | 60.0 | 64.0 | 59.1 | 56.8 | 57.3 | 61.3 | 53.6 | 51.5 | 50.0           | 51.5 | 58.18 | 58.17 |

<sup>1</sup> No comparisons for model fitting were made because no model fitting forecasts were provided for Box-Jenkins in the present study.<sup>2</sup> The method of A.E.P. Filtering used in the current study is somewhat different (see description of A.E.P. method) from that used in the study reported in JRSS.

were seasonal or not. Thus, while some methods (e.g. deseasonalised single exponential smoothing) perform well for monthly data, they may do badly for, say, yearly data. Tables 12, 13, 14, 15, 16, 17 and 18 show the MAPE for yearly, quarterly, monthly, micro, macro, non-seasonal and seasonal data. Tables 19, 20, 21, 22, 23, 24 and 25 do so for the average rankings, whereas Tables 26, 27, 28, 29, 30, 31 and 32 do so for the medians.

It is to be expected that methods which do not take trend into account will not do as well as methods which do for data subject to substantial trends (e.g. yearly). Single exponential smoothing does not do very well therefore, whereas Holt or Holt-Winters (for yearly data the two are equivalent) and Lewandowski do the best. Single exponential smoothing is progressively worse as the time horizon increases, precisely because it does not take trend into account. Bayesian forecasting and the Box-Jenkins method do about the same as single exponential smoothing (the reason could be that the trend is over-extended in the forecasting). For monthly data, deseasonalized single exponential smoothing does relatively better than Holt-Winters, Automatic AEP, Bayesian forecasting, Box-Jenkins and Lewandowski.

The most striking differences are between micro and macro data (see Tables 15, 16, 22 and 23). In micro data the simple methods do much better than the statistically sophisticated methodologies, which, in turn, are at their best with macro data. For instance, the overall MAPE for Lewandowski is 13.7% for micro and 18.2% for macro, whereas that of Parzen is 18.4% for micro and 11.2% for macro. Even for the small number of series in each category (33 micro and 35 macro) these differences are significant.

Finally, it is interesting to note that for seasonal data, deseasonalized single and adaptive response rate, exponential smoothing, deseasonalized regression, Bayesian forecasting and Parzen do about the same as far as overall MAPE is concerned. For non-seasonal data the MAPEs are much more spread out as sophisticated methods do relatively better than with seasonal data. Furthermore, the differences in overall average ranking for the various methods are even more pronounced for non-seasonal data, whereas they (excluding non-seasonal methods) are very small for seasonal data.

It seems that the factors affecting forecasting accuracy are trend, seasonality and randomness (noise) present in the data. It is believed that the greater the randomness in the data, the less important is the use of statistically sophisticated methods. Furthermore, it seems that deseasonalizing the data by a simple decomposition procedure is adequate, making the majority of methods (both simple and sophisticated) perform about the same. Finally, it is believed that some statistically sophisticated methods extrapolate too much trend which can cause overestimation. This is why Naive 2 and single exponential smoothing do relatively well in comparison to some statistically sophisticated methods.

### Effects of forecasting horizons

For short forecasting horizons (1 and 2 periods ahead) deseasonalized simple, Holt, Brown and Holt-Winters exponential smoothing do well. For horizons 3, 4, 5 and 6 deseasonalized Holt, Brown, and Holt-Winters, and Parzen perform relatively well in most accuracy criteria. Finally, for longer time horizons (i.e. 7, 8, 9, ..., 18) Lewandowski does the best.

### The combining of forecasts

Combining A, a simple average of six methods (see Appendix 2), performs very well overall and better than the individual methods included in the average.

Combining B (using the same methods as Combining A but taking a weighted average based on the sample covariance matrix of fitting errors—instead of the simple average of Combining A) also performs well, but not as well as Combining A.

Table 12. Average MAPE: yearly data (20)

| METHODS     | MODEL<br>FITTING | Forecasting Horizons |      |      |      |      |      |     |     |     |     | Average of Forecasting Horizons |      |      |      |      | n(max) |      |
|-------------|------------------|----------------------|------|------|------|------|------|-----|-----|-----|-----|---------------------------------|------|------|------|------|--------|------|
|             |                  | 1                    | 2    | 3    | 4    | 5    | 6    | 8   | 12  | 15  | 18  | 1-4                             | 1-6  | 1-8  | 1-12 | 1-15 |        | 1-18 |
| NAIVE 1     | 10.9             | 6.8                  | 9.7  | 16.6 | 21.1 | 23.8 | 24.8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 13.6 | 17.1 | 17.1 | 17.1 | 17.1   | 20   |
| Mov.Averag  | 10.7             | 8.6                  | 10.9 | 17.7 | 21.9 | 24.7 | 26.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 13.8 | 18.3 | 18.3 | 18.3 | 18.3   | 20   |
| Single Exp  | 11.4             | 6.2                  | 9.1  | 16.3 | 21.0 | 23.6 | 25.4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 13.1 | 16.9 | 16.9 | 16.9 | 16.9   | 20   |
| MR EXP      | 13.4             | 7.8                  | 13.7 | 17.7 | 24.4 | 25.3 | 29.3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 15.9 | 19.7 | 19.7 | 19.7 | 19.7   | 20   |
| Holt EXP    | 12.9             | 5.6                  | 7.2  | 11.9 | 16.2 | 19.0 | 16.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 10.2 | 12.7 | 12.7 | 12.7 | 12.7   | 20   |
| Brown EXP   | 10.8             | 6.7                  | 8.2  | 12.0 | 16.5 | 19.8 | 16.4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 10.8 | 13.3 | 13.3 | 13.3 | 13.3   | 20   |
| Quad-EXP    | 10.6             | 7.0                  | 8.6  | 11.8 | 16.0 | 20.7 | 17.4 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 10.9 | 13.6 | 13.6 | 13.6 | 13.6   | 20   |
| Regression  | 8.6              | 6.9                  | 7.8  | 14.9 | 18.4 | 20.0 | 20.6 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 12.0 | 14.8 | 14.8 | 14.8 | 14.8   | 19   |
| WINTERS     | 12.9             | 5.6                  | 7.2  | 11.9 | 16.2 | 19.0 | 16.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 10.2 | 12.7 | 12.7 | 12.7 | 12.7   | 20   |
| Auton. AEP  | 8.7              | 7.1                  | 8.9  | 11.1 | 17.8 | 21.8 | 19.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 11.9 | 18.8 | 18.8 | 18.8 | 18.8   | 20   |
| Bayesian F  | 20.2             | 12.2                 | 12.6 | 16.9 | 18.0 | 20.6 | 20.6 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 18.4 | 16.5 | 16.5 | 16.5 | 16.5   | 20   |
| Combining A | 9.4              | 5.7                  | 7.7  | 12.5 | 17.4 | 20.0 | 17.8 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 10.8 | 13.3 | 13.3 | 13.3 | 13.3   | 20   |
| Combining B | 9.0              | 6.3                  | 8.3  | 13.7 | 17.5 | 19.7 | 20.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 11.5 | 14.3 | 14.3 | 14.3 | 14.3   | 20   |
| Box-Jenkins | 9.4              | 7.2                  | 10.8 | 13.7 | 18.6 | 23.2 | 22.3 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 12.6 | 16.0 | 16.0 | 16.0 | 16.0   | 20   |
| Lewandowski | 20.1             | 7.3                  | 8.3  | 16.7 | 13.8 | 16.8 | 15.1 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 11.0 | 12.7 | 12.7 | 12.7 | 12.7   | 20   |
| Parzen      | 9.6              | 7.6                  | 7.7  | 12.8 | 16.0 | 20.5 | 18.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 11.0 | 13.8 | 13.8 | 13.8 | 13.8   | 20   |
| Average     | 11.1             | 7.1                  | 9.2  | 14.4 | 18.6 | 21.5 | 20.9 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0                             | 12.3 | 15.3 | 15.3 | 15.3 | 15.3   | 20   |

Table 13. Average MAPE: quarterly data (23)

| METHODS     | MODEL<br>FITTING | Forecasting Horizons |      |      |      |      |      |      |     | Average of Forecasting Horizons |     |     |      |      | n(max) |      |      |      |
|-------------|------------------|----------------------|------|------|------|------|------|------|-----|---------------------------------|-----|-----|------|------|--------|------|------|------|
|             |                  | 1                    | 2    | 3    | 4    | 5    | 6    | 8    | 12  | 15                              | 18  | 1-4 | 1-6  | 1-8  |        | 1-12 | 1-15 | 1-18 |
| NAIVE 1     | 11.3             | 8.5                  | 11.2 | 19.2 | 21.5 | 25.6 | 25.2 | 23.4 | 0.0 | 0.0                             | 0.0 | 0.0 | 15.1 | 18.5 | 20.9   | 20.9 | 20.9 | 23   |
| Mov.Averag  | 9.3              | 16.3                 | 18.9 | 24.3 | 28.5 | 32.9 | 32.6 | 31.0 | 0.0 | 0.0                             | 0.0 | 0.0 | 22.0 | 23.6 | 28.1   | 28.1 | 28.1 | 23   |
| Single Exp  | 9.6              | 10.0                 | 11.7 | 16.8 | 21.3 | 24.6 | 25.1 | 23.2 | 0.0 | 0.0                             | 0.0 | 0.0 | 14.9 | 18.3 | 20.6   | 20.6 | 20.6 | 23   |
| ARR EXP     | 11.4             | 13.6                 | 16.7 | 19.3 | 22.9 | 26.2 | 24.6 | 24.1 | 0.0 | 0.0                             | 0.0 | 0.0 | 18.1 | 20.6 | 22.4   | 22.4 | 22.4 | 23   |
| Holt EXP    | 9.1              | 10.0                 | 10.3 | 21.1 | 25.9 | 34.3 | 37.3 | 41.9 | 0.0 | 0.0                             | 0.0 | 0.0 | 16.8 | 23.1 | 29.0   | 29.0 | 29.0 | 23   |
| Brown EXP   | 9.3              | 11.3                 | 10.0 | 19.6 | 26.6 | 36.1 | 39.3 | 44.9 | 0.0 | 0.0                             | 0.0 | 0.0 | 16.9 | 23.8 | 30.1   | 30.1 | 30.1 | 23   |
| Quad-EXP    | 9.4              | 12.3                 | 13.3 | 26.2 | 36.7 | 49.2 | 58.0 | 79.3 | 0.0 | 0.0                             | 0.0 | 0.0 | 22.1 | 32.6 | 43.7   | 43.7 | 43.7 | 23   |
| Regression  | 12.1             | 20.1                 | 20.2 | 22.2 | 25.9 | 31.0 | 35.2 | 23.9 | 0.0 | 0.0                             | 0.0 | 0.0 | 22.1 | 24.1 | 25.5   | 25.5 | 25.5 | 23   |
| NAIVE2      | 9.0              | 7.6                  | 12.0 | 15.8 | 21.5 | 22.3 | 22.3 | 23.3 | 0.0 | 0.0                             | 0.0 | 0.0 | 14.2 | 16.9 | 19.0   | 19.0 | 19.0 | 23   |
| D Mov.Aver3 | 7.7              | 14.4                 | 18.3 | 23.2 | 27.4 | 30.6 | 31.3 | 29.5 | 0.0 | 0.0                             | 0.0 | 0.0 | 20.8 | 22.2 | 26.5   | 26.5 | 26.5 | 23   |
| D Sing EXP  | 7.7              | 9.0                  | 12.0 | 14.4 | 20.5 | 21.0 | 21.9 | 22.6 | 0.0 | 0.0                             | 0.0 | 0.0 | 18.0 | 18.5 | 18.5   | 18.5 | 18.5 | 23   |
| D ARR EXP   | 9.6              | 12.3                 | 16.8 | 18.2 | 25.0 | 25.3 | 24.3 | 26.0 | 0.0 | 0.0                             | 0.0 | 0.0 | 18.1 | 20.3 | 22.2   | 22.2 | 22.2 | 23   |
| D Holt EXP  | 7.2              | 9.2                  | 10.4 | 17.1 | 25.1 | 30.3 | 32.2 | 39.2 | 0.0 | 0.0                             | 0.0 | 0.0 | 15.4 | 20.7 | 25.9   | 25.9 | 25.9 | 23   |
| D BrownEXP  | 7.3              | 10.0                 | 10.4 | 15.1 | 22.5 | 27.1 | 30.5 | 36.5 | 0.0 | 0.0                             | 0.0 | 0.0 | 14.5 | 19.3 | 24.0   | 24.0 | 24.0 | 23   |
| D Quad-EXP  | 7.6              | 11.1                 | 12.5 | 21.1 | 32.0 | 39.2 | 46.0 | 66.6 | 0.0 | 0.0                             | 0.0 | 0.0 | 19.2 | 27.0 | 35.6   | 35.6 | 35.6 | 23   |
| D Regress   | 11.5             | 18.1                 | 21.2 | 22.4 | 26.3 | 28.6 | 24.5 | 25.2 | 0.0 | 0.0                             | 0.0 | 0.0 | 22.0 | 23.5 | 24.8   | 24.8 | 24.8 | 23   |
| WINTERS     | 7.3              | 8.9                  | 9.1  | 17.1 | 25.6 | 32.6 | 37.2 | 40.3 | 0.0 | 0.0                             | 0.0 | 0.0 | 13.2 | 20.9 | 26.4   | 26.4 | 26.4 | 23   |
| Auton. AEP  | 16.5             | 8.3                  | 8.8  | 15.4 | 22.4 | 27.2 | 34.7 | 40.2 | 0.0 | 0.0                             | 0.0 | 0.0 | 13.7 | 19.8 | 25.9   | 25.9 | 25.9 | 23   |
| Bayesian F  | 10.4             | 12.7                 | 18.6 | 20.4 | 24.7 | 27.8 | 26.8 | 28.8 | 0.0 | 0.0                             | 0.0 | 0.0 | 15.1 | 21.8 | 24.6   | 24.6 | 24.6 | 23   |
| Combining A | 8.8              | 8.3                  | 8.0  | 11.7 | 19.4 | 24.4 | 26.3 | 31.0 | 0.0 | 0.0                             | 0.0 | 0.0 | 11.8 | 16.3 | 20.7   | 20.7 | 20.7 | 23   |
| Combining B | 10.3             | 8.5                  | 10.1 | 13.9 | 23.6 | 26.7 | 27.7 | 33.5 | 0.0 | 0.0                             | 0.0 | 0.0 | 14.0 | 18.4 | 22.4   | 22.4 | 22.4 | 23   |
| Box-Jenkins | 9.8              | 7.6                  | 8.2  | 13.9 | 21.3 | 26.1 | 26.1 | 26.4 | 0.0 | 0.0                             | 0.0 | 0.0 | 13.7 | 17.2 | 20.1   | 20.1 | 20.1 | 23   |
| Lewandowski | 9.8              | 12.5                 | 14.1 | 14.2 | 21.8 | 24.8 | 22.8 | 26.9 | 0.0 | 0.0                             | 0.0 | 0.0 | 15.7 | 18.4 | 20.6   | 20.6 | 20.6 | 23   |
| Parzen      | 7.7              | 6.8                  | 7.6  | 12.0 | 16.5 | 21.1 | 20.4 | 21.0 | 0.0 | 0.0                             | 0.0 | 0.0 | 10.7 | 14.1 | 16.7   | 16.7 | 16.7 | 23   |
| Average     | 9.3              | 11.1                 | 12.0 | 15.1 | 24.4 | 27.0 | 29.0 | 33.6 | 0.0 | 0.0                             | 0.0 | 0.0 | 16.6 | 20.9 | 24.8   | 24.8 | 24.8 | 23   |

Table 14. Average MAPE: monthly data (68)

| METHODS     | MODEL FITTING | Forecasting Horizons |      |      |      |      |      |      |      |       |       |      |      | Average of Forecasting Horizons |      |      |             |
|-------------|---------------|----------------------|------|------|------|------|------|------|------|-------|-------|------|------|---------------------------------|------|------|-------------|
|             |               | 1                    | 2    | 3    | 4    | 5    | 6    | 8    | 12   | 15    | 18    | 1-4  | 1-6  | 1-8                             | 1-12 | 1-15 | 1-18 n(max) |
| NAIVE 1     | 16.5          | 16.7                 | 21.6 | 21.4 | 16.9 | 21.0 | 22.6 | 28.3 | 14.5 | 31.9  | 34.9  | 19.1 | 20.0 | 21.4                            | 20.0 | 21.4 | 23.0        |
| Mov.Aver3   | 14.5          | 15.0                 | 18.0 | 17.7 | 14.8 | 17.2 | 19.9 | 21.4 | 16.3 | 28.7  | 31.9  | 16.4 | 17.1 | 17.7                            | 16.9 | 19.3 | 68          |
| Single Exp  | 15.0          | 14.7                 | 17.6 | 17.9 | 15.4 | 17.9 | 20.8 | 22.5 | 16.1 | 28.8  | 32.5  | 16.4 | 17.3 | 18.2                            | 17.3 | 18.7 | 20.3        |
| ARR Exp     | 16.8          | 14.4                 | 18.2 | 18.4 | 14.9 | 17.5 | 20.3 | 21.9 | 16.1 | 29.6  | 32.2  | 16.5 | 17.4 | 18.0                            | 17.2 | 18.7 | 68          |
| Holt Exp    | 15.3          | 14.9                 | 17.1 | 18.1 | 17.8 | 20.5 | 23.3 | 27.5 | 22.6 | 40.4  | 40.3  | 17.0 | 18.6 | 20.3                            | 20.8 | 23.7 | 68          |
| Brown Exp   | 15.4          | 15.4                 | 18.8 | 20.2 | 19.0 | 23.1 | 26.2 | 31.7 | 28.0 | 54.0  | 59.6  | 18.1 | 20.3 | 22.5                            | 23.6 | 27.8 | 68          |
| Quads Exp   | 16.4          | 15.4                 | 19.2 | 23.4 | 20.8 | 26.8 | 31.0 | 42.1 | 49.0 | 103.1 | 106.0 | 19.7 | 22.8 | 26.5                            | 31.6 | 43.0 | 68          |
| Regression  | 20.3          | 20.2                 | 23.1 | 22.5 | 20.4 | 21.4 | 26.2 | 27.0 | 26.1 | 49.5  | 60.8  | 21.6 | 22.3 | 23.2                            | 23.4 | 26.6 | 68          |
| NAIVE2      | 8.6           | 9.2                  | 11.7 | 12.4 | 11.7 | 12.6 | 13.5 | 16.0 | 14.5 | 31.2  | 30.2  | 11.3 | 11.8 | 13.0                            | 13.7 | 15.8 | 17.7        |
| D Mov.Aver3 | 7.5           | 10.1                 | 12.8 | 16.0 | 16.0 | 18.2 | 19.4 | 20.4 | 15.7 | 28.3  | 34.0  | 13.7 | 15.4 | 16.6                            | 16.6 | 17.8 | 68          |
| D Sing Exp  | 9.0           | 7.9                  | 10.9 | 11.7 | 10.6 | 11.6 | 13.2 | 14.4 | 13.6 | 29.3  | 30.1  | 10.3 | 11.0 | 12.0                            | 12.6 | 14.5 | 68          |
| D Holt Exp  | 7.9           | 8.2                  | 11.5 | 12.3 | 11.4 | 12.5 | 15.2 | 17.7 | 16.5 | 35.6  | 35.2  | 10.9 | 11.8 | 13.5                            | 14.8 | 17.2 | 68          |
| D brown Exp | 9.0           | 8.4                  | 11.6 | 13.0 | 11.7 | 13.3 | 16.4 | 19.6 | 19.0 | 43.1  | 45.4  | 11.3 | 12.3 | 14.2                            | 16.0 | 19.5 | 68          |
| D Quads Exp | 9.0           | 8.6                  | 12.5 | 13.4 | 12.1 | 16.3 | 18.7 | 25.3 | 29.7 | 56.1  | 61.6  | 11.7 | 13.7 | 16.6                            | 20.4 | 25.7 | 68          |
| D Regress   | 13.1          | 12.2                 | 14.7 | 16.0 | 15.7 | 16.6 | 19.9 | 19.6 | 23.4 | 48.5  | 57.3  | 14.7 | 15.9 | 16.8                            | 18.1 | 21.4 | 68          |
| WINTERS     | 9.0           | 10.3                 | 12.0 | 12.5 | 11.8 | 11.9 | 14.8 | 17.5 | 15.9 | 33.4  | 34.5  | 11.7 | 12.2 | 13.6                            | 14.6 | 16.8 | 68          |
| Autom. ARP  | 9.5           | 11.2                 | 12.8 | 13.0 | 11.9 | 11.2 | 13.4 | 17.6 | 16.2 | 30.2  | 33.9  | 12.2 | 12.2 | 13.4                            | 14.2 | 16.1 | 68          |
| Bayesian F  | 12.1          | 8.9                  | 10.8 | 10.8 | 9.9  | 10.9 | 12.8 | 16.0 | 16.1 | 27.5  | 30.6  | 10.1 | 10.7 | 11.8                            | 12.6 | 14.5 | 68          |
| Combining A | 7.8           | 8.4                  | 11.1 | 11.8 | 10.4 | 11.1 | 13.4 | 15.6 | 14.2 | 32.4  | 33.3  | 10.4 | 11.0 | 12.3                            | 13.1 | 15.3 | 68          |
| Combining B | 7.5           | 8.6                  | 10.7 | 10.6 | 10.8 | 10.2 | 11.5 | 15.6 | 15.5 | 31.3  | 31.4  | 10.2 | 10.4 | 11.7                            | 13.0 | 15.3 | 68          |
| Box-Jenkins | N/A           | 12.1                 | 11.5 | 9.9  | 11.1 | 11.0 | 12.5 | 16.7 | 16.4 | 26.2  | 34.2  | 11.1 | 11.3 | 12.7                            | 13.8 | 15.6 | 68          |
| Lewandowski | 10.8          | 12.6                 | 13.6 | 14.6 | 13.5 | 13.8 | 16.6 | 16.2 | 17.0 | 33.0  | 28.6  | 13.6 | 14.1 | 14.9                            | 17.1 | 18.9 | 68          |
| Parzen      | 9.0           | 12.7                 | 12.6 | 9.6  | 11.7 | 10.2 | 11.8 | 14.3 | 13.7 | 22.5  | 26.5  | 11.7 | 11.4 | 12.1                            | 12.6 | 13.9 | 68          |
| Average     | 11.1          | 11.8                 | 14.4 | 15.0 | 13.7 | 15.3 | 17.7 | 20.7 | 19.2 | 37.5  | 40.7  | 13.7 | 14.7 | 16.0                            | 16.8 | 19.6 | 22.3        |

Table 15. Average MAPE: micro data (33)

| METHODS     | MODEL FITTING | Forecasting Horizons |      |      |      |      |      |      |      |      |      |      |      | Average of Forecasting Horizons |      |      |             |
|-------------|---------------|----------------------|------|------|------|------|------|------|------|------|------|------|------|---------------------------------|------|------|-------------|
|             |               | 1                    | 2    | 3    | 4    | 5    | 6    | 8    | 12   | 15   | 18   | 1-4  | 1-6  | 1-8                             | 1-12 | 1-15 | 1-18 n(max) |
| NAIVE 1     | 19.7          | 17.7                 | 25.4 | 31.3 | 25.9 | 33.4 | 25.0 | 17.8 | 19.7 | 38.9 | 31.3 | 25.1 | 26.5 | 25.6                            | 24.5 | 25.7 | 26.8        |
| Mov.Aver3   | 18.1          | 15.0                 | 19.9 | 24.7 | 21.6 | 28.3 | 20.7 | 10.8 | 22.0 | 31.1 | 24.5 | 20.3 | 21.7 | 20.7                            | 19.8 | 20.6 | 21.4        |
| Single Exp  | 18.4          | 14.8                 | 19.0 | 24.0 | 22.7 | 28.5 | 21.6 | 10.7 | 21.1 | 30.1 | 25.4 | 20.1 | 21.8 | 20.6                            | 19.8 | 20.5 | 21.6        |
| ARR Exp     | 20.3          | 15.0                 | 20.3 | 24.9 | 21.7 | 27.0 | 20.7 | 9.8  | 19.9 | 28.2 | 22.9 | 20.4 | 21.5 | 20.2                            | 19.2 | 19.6 | 20.4        |
| Holt Exp    | 19.5          | 15.8                 | 20.2 | 26.6 | 22.6 | 32.0 | 22.1 | 15.8 | 22.0 | 41.9 | 33.1 | 21.3 | 23.2 | 23.0                            | 22.3 | 23.6 | 25.6        |
| Brown Exp   | 19.5          | 18.1                 | 22.9 | 27.6 | 22.6 | 35.5 | 24.0 | 16.9 | 25.5 | 49.5 | 43.0 | 22.8 | 25.1 | 24.8                            | 24.3 | 26.6 | 29.6        |
| Quads Exp   | 19.0          | 18.5                 | 23.6 | 31.0 | 25.6 | 39.1 | 28.7 | 29.1 | 41.5 | 82.2 | 85.5 | 24.7 | 27.7 | 28.5                            | 30.5 | 36.3 | 43.1        |
| Regression  | 20.3          | 17.7                 | 20.1 | 25.5 | 21.1 | 27.3 | 15.6 | 12.4 | 18.9 | 28.5 | 17.8 | 21.1 | 21.9 | 21.2                            | 19.7 | 19.8 | 20.2        |
| NAIVE2      | 14.3          | 12.4                 | 19.7 | 17.9 | 17.0 | 22.9 | 19.4 | 14.9 | 19.7 | 28.8 | 26.3 | 16.8 | 18.2 | 18.5                            | 18.5 | 19.6 | 22.4        |
| D Mov.Aver3 | 13.0          | 12.6                 | 17.6 | 20.2 | 18.1 | 25.6 | 22.1 | 17.4 | 20.1 | 29.2 | 32.0 | 17.1 | 19.4 | 20.2                            | 20.1 | 21.0 | 22.2        |
| D Sing Exp  | 13.6          | 11.8                 | 17.6 | 15.8 | 15.1 | 19.9 | 18.0 | 11.5 | 17.4 | 24.1 | 25.3 | 15.1 | 16.4 | 16.5                            | 16.3 | 17.2 | 18.0        |
| D ARR Exp   | 15.7          | 11.6                 | 17.4 | 15.1 | 14.9 | 19.5 | 17.5 | 10.9 | 18.3 | 23.3 | 23.6 | 14.8 | 16.0 | 16.0                            | 16.0 | 16.7 | 17.3        |
| D Holt Exp  | 13.9          | 14.2                 | 19.9 | 18.9 | 15.4 | 25.1 | 21.7 | 18.4 | 22.7 | 41.7 | 46.0 | 16.4 | 17.9 | 19.0                            | 18.6 | 19.9 | 21.3        |
| D brown Exp | 14.7          | 14.8                 | 21.5 | 20.0 | 16.0 | 29.8 | 26.6 | 29.6 | 39.4 | 69.4 | 82.7 | 18.1 | 21.5 | 23.8                            | 27.0 | 32.5 | 39.0        |
| D Quads Exp | 15.6          | 11.9                 | 17.1 | 17.8 | 15.9 | 20.8 | 17.5 | 12.4 | 13.2 | 23.2 | 22.2 | 15.7 | 16.8 | 17.0                            | 15.6 | 15.9 | 16.7        |
| D Regress   | 15.8          | 14.9                 | 19.7 | 16.7 | 16.3 | 23.0 | 19.1 | 15.0 | 17.6 | 29.3 | 34.5 | 17.4 | 18.6 | 19.3                            | 18.9 | 20.0 | 21.7        |
| WINTERS     | 14.0          | 18.0                 | 19.9 | 17.7 | 16.5 | 21.2 | 13.7 | 12.2 | 12.1 | 28.2 | 30.1 | 18.1 | 17.8 | 18.1                            | 18.4 | 19.6 | 20.9        |
| Autom. ARP  | 14.8          | 13.7                 | 19.4 | 17.1 | 15.0 | 20.5 | 18.8 | 18.2 | 22.6 | 36.4 | 37.4 | 15.3 | 17.4 | 18.8                            | 18.9 | 20.7 | 22.7        |
| Bayesian F  | 12.3          | 13.3                 | 18.2 | 16.6 | 14.8 | 21.4 | 17.0 | 13.1 | 16.1 | 29.6 | 30.7 | 15.7 | 16.9 | 17.4                            | 17.2 | 18.3 | 19.6        |
| Combining A | 11.6          | 12.9                 | 16.9 | 13.5 | 14.8 | 16.6 | 11.9 | 10.3 | 16.1 | 24.2 | 22.4 | 14.5 | 14.8 | 14.9                            | 15.0 | 16.0 | 16.6        |
| Combining B | N/A           | 17.1                 | 19.8 | 14.1 | 17.4 | 22.7 | 14.5 | 12.6 | 18.6 | 25.9 | 28.7 | 17.1 | 17.6 | 17.7                            | 17.7 | 18.9 | 20.2        |
| Box-Jenkins | N/A           | 13.2                 | 14.4 | 12.1 | 11.6 | 13.9 | 13.3 | 10.9 | 12.6 | 17.8 | 17.3 | 12.8 | 13.1 | 13.5                            | 13.7 | 13.8 | 13.7        |
| Lewandowski | 20.3          | 14.8                 | 16.9 | 21.4 | 18.1 | 16.7 | 18.1 | 14.1 | 11.9 | 17.2 | 26.2 | 17.5 | 17.8 | 17.2                            | 17.0 | 16.8 | 18.4        |
| Parzen      | 14.8          | 15.8                 | 19.6 | 20.1 | 18.1 | 24.8 | 19.5 | 15.0 | 20.9 | 34.3 | 33.4 | 18.2 | 19.5 | 19.7                            | 19.6 | 21.0 | 22.7        |
| Average     | 15.8          | 14.9                 | 19.6 | 20.1 | 18.1 | 24.8 | 19.5 | 15.0 | 20.9 | 34.3 | 33.4 | 18.2 | 19.5 | 19.7                            | 19.6 | 21.0 | 22.7        |

Table 16. Average MAPE: macro data (35)

| METHODS       | MODEL FITTING | Forecasting Horizons |      |      |      |      |      |      |      |      |      | Average of Forecasting Horizons |      |      |      |      |      | n(max) |
|---------------|---------------|----------------------|------|------|------|------|------|------|------|------|------|---------------------------------|------|------|------|------|------|--------|
|               |               | 1                    | 2    | 3    | 4    | 5    | 6    | 8    | 12   | 15   | 18   | 1-4                             | 1-6  | 1-8  | 1-12 | 1-15 | 1-18 |        |
| NAIVE 1       | 7.7           | 6.8                  | 10.5 | 10.5 | 12.8 | 14.9 | 15.4 | 16.2 | 13.1 | 32.1 | 33.8 | 10.1                            | 11.0 | 12.8 | 13.0 | 14.5 | 15.9 | 35     |
| Mov.Averag    | 6.7           | 8.5                  | 12.4 | 11.8 | 15.8 | 17.3 | 18.6 | 16.6 | 14.9 | 31.5 | 33.4 | 12.1                            | 14.1 | 14.7 | 14.3 | 15.5 | 16.7 | 35     |
| Single Exp    | 6.5           | 5.9                  | 9.9  | 9.5  | 13.2 | 14.6 | 16.1 | 13.7 | 15.6 | 32.0 | 33.8 | 9.6                             | 11.5 | 12.1 | 12.4 | 13.9 | 15.3 | 35     |
| ARR Exp       | 8.1           | 5.5                  | 11.0 | 10.1 | 14.6 | 16.6 | 17.8 | 15.9 | 17.3 | 36.7 | 38.5 | 10.3                            | 12.3 | 13.1 | 13.5 | 15.3 | 17.1 | 35     |
| Holt Exp      | 5.4           | 5.2                  | 8.0  | 6.7  | 10.0 | 11.9 | 13.3 | 14.0 | 16.6 | 32.1 | 34.0 | 7.5                             | 9.2  | 10.3 | 10.9 | 12.7 | 14.3 | 35     |
| Brown Exp     | 6.2           | 5.5                  | 8.5  | 6.1  | 11.8 | 14.2 | 15.9 | 19.0 | 30.2 | 64.8 | 79.6 | 8.5                             | 10.7 | 12.4 | 14.9 | 19.4 | 24.3 | 35     |
| QuadsExp      | 6.3           | 4.9                  | 7.8  | 7.0  | 10.9 | 13.4 | 15.5 | 21.4 | 24.9 | 60.7 | 86.2 | 7.6                             | 9.9  | 12.1 | 13.6 | 17.8 | 23.3 | 35     |
| Regression    | 8.4           | 9.8                  | 12.8 | 11.4 | 14.4 | 15.0 | 15.8 | 16.1 | 16.1 | 29.5 | 30.4 | 12.1                            | 13.2 | 13.8 | 14.0 | 15.3 | 16.3 | 35     |
| NAIVE2        | 4.9           | 4.1                  | 7.4  | 11.4 | 13.7 | 13.2 | 16.1 | 14.0 | 13.1 | 52.2 | 39.2 | 9.2                             | 11.0 | 11.7 | 12.5 | 15.3 | 17.4 | 35     |
| D Mov.Averag  | 4.4           | 8.2                  | 10.1 | 13.5 | 17.1 | 17.6 | 19.8 | 17.7 | 12.9 | 34.7 | 34.0 | 12.2                            | 14.4 | 15.0 | 14.9 | 16.0 | 17.6 | 35     |
| D Sing Exp    | 4.4           | 4.3                  | 7.6  | 11.4 | 13.8 | 13.2 | 16.4 | 13.9 | 13.0 | 52.0 | 39.1 | 9.3                             | 11.1 | 11.8 | 12.5 | 15.3 | 17.4 | 35     |
| D ARR Exp     | 5.5           | 4.8                  | 8.9  | 11.1 | 15.0 | 13.3 | 17.3 | 14.2 | 12.7 | 48.8 | 37.1 | 10.0                            | 11.8 | 12.3 | 12.8 | 15.3 | 17.2 | 35     |
| D Holt Exp    | 3.6           | 3.4                  | 5.8  | 4.4  | 10.7 | 10.2 | 13.6 | 16.0 | 15.7 | 63.6 | 45.8 | 7.0                             | 8.7  | 10.1 | 11.5 | 15.2 | 18.0 | 35     |
| D BrownExp    | 3.8           | 3.5                  | 5.8  | 4.8  | 10.9 | 10.3 | 14.0 | 15.9 | 15.6 | 71.4 | 53.9 | 7.3                             | 8.9  | 10.3 | 11.9 | 16.3 | 19.6 | 35     |
| D QuadsExp    | 3.9           | 3.6                  | 5.9  | 8.5  | 11.2 | 10.9 | 15.1 | 20.3 | 12.1 | 56.5 | 42.8 | 7.3                             | 9.2  | 11.3 | 11.9 | 15.2 | 17.6 | 35     |
| D Regression  | 7.1           | 9.3                  | 9.1  | 10.9 | 13.1 | 13.7 | 15.1 | 13.7 | 13.1 | 34.9 | 25.1 | 10.6                            | 11.9 | 12.2 | 12.4 | 14.0 | 18.9 | 35     |
| WINTERS       | 4.1           | 5.2                  | 5.2  | 9.0  | 11.8 | 11.1 | 13.6 | 15.7 | 14.6 | 68.7 | 54.4 | 7.8                             | 8.3  | 10.6 | 11.9 | 15.7 | 19.4 | 35     |
| Auton. AFP    | 9.4           | 5.9                  | 6.0  | 8.3  | 10.8 | 10.6 | 11.4 | 14.2 | 14.2 | 48.6 | 37.4 | 7.8                             | 8.8  | 9.8  | 11.2 | 13.8 | 16.2 | 35     |
| Bavesian F    | 8.6           | 6.2                  | 7.7  | 9.4  | 10.6 | 12.3 | 13.5 | 12.2 | 13.2 | 31.5 | 22.7 | 8.5                             | 9.9  | 10.3 | 10.6 | 12.1 | 12.9 | 35     |
| Comolininga A | 4.7           | 4.0                  | 5.9  | 6.8  | 11.4 | 10.4 | 13.2 | 13.5 | 13.2 | 57.9 | 43.0 | 7.5                             | 8.9  | 9.9  | 11.0 | 14.3 | 16.8 | 35     |
| Comolininga B | 5.9           | 4.8                  | 6.8  | 10.1 | 12.1 | 11.2 | 13.4 | 12.5 | 15.4 | 50.9 | 38.6 | 8.5                             | 9.7  | 10.4 | 11.1 | 14.1 | 16.1 | 35     |
| Box-Jenkins   | 4.4           | 6.0                  | 5.7  | 6.5  | 10.5 | 10.3 | 12.4 | 16.0 | 16.2 | 36.3 | 35.5 | 7.2                             | 8.6  | 10.0 | 11.6 | 13.5 | 15.3 | 35     |
| Levan-Jenski  | 5.3           | 6.6                  | 10.9 | 11.5 | 14.0 | 11.8 | 15.8 | 14.4 | 16.1 | 61.0 | 36.6 | 10.7                            | 11.8 | 12.0 | 12.5 | 16.4 | 18.2 | 35     |
| Parzen        | 4.0           | 4.9                  | 4.6  | 5.5  | 9.0  | 9.6  | 9.2  | 10.8 | 10.9 | 26.5 | 25.3 | 6.0                             | 7.1  | 7.9  | 8.4  | 9.9  | 11.2 | 35     |
| Average       | 5.7           | 5.7                  | 8.1  | 9.5  | 17.5 | 12.7 | 14.9 | 15.1 | 15.3 | 46.5 | 40.8 | 8.9                             | 10.6 | 11.5 | 12.3 | 14.9 | 17.0 |        |

Table 17. Average MAPE: seasonal data (60)

| METHODS       | MODEL FITTING | Forecasting Horizons |      |      |      |      |      |      |      |       |      | Average of Forecasting Horizons |      |      |      |      | n(max) |      |
|---------------|---------------|----------------------|------|------|------|------|------|------|------|-------|------|---------------------------------|------|------|------|------|--------|------|
|               |               | 1                    | 2    | 3    | 4    | 5    | 6    | 8    | 12   | 15    | 18   | 1-4                             | 1-6  | 1-8  | 1-12 | 1-15 |        | 1-18 |
| NAIVE 1       | 17.8          | 10.1                 | 20.7 | 21.9 | 19.3 | 22.1 | 24.8 | 32.2 | 13.0 | 30.7  | 31.2 | 19.5                            | 20.8 | 22.9 | 21.3 | 22.2 | 22.9   | 60   |
| Mov.Averag    | 15.2          | 14.7                 | 17.5 | 17.3 | 17.2 | 19.5 | 22.4 | 25.1 | 15.6 | 27.0  | 27.8 | 16.7                            | 17.9 | 19.1 | 18.0 | 18.9 | 19.5   | 60   |
| Single Exp    | 15.7          | 14.1                 | 17.2 | 17.4 | 17.8 | 19.1 | 23.1 | 26.3 | 15.6 | 27.1  | 28.7 | 16.7                            | 18.2 | 19.6 | 18.5 | 19.3 | 20.0   | 60   |
| ARR Exp       | 17.5          | 14.7                 | 19.0 | 18.1 | 16.1 | 18.2 | 21.7 | 25.0 | 15.4 | 28.3  | 28.8 | 17.0                            | 18.0 | 19.2 | 18.2 | 19.2 | 20.0   | 60   |
| Holt Exp      | 16.0          | 14.5                 | 15.1 | 18.3 | 20.6 | 21.9 | 26.2 | 31.8 | 23.8 | 43.3  | 40.3 | 17.4                            | 19.6 | 22.0 | 22.3 | 25.0 | 26.8   | 60   |
| Brown Exp     | 16.7          | 14.4                 | 17.5 | 19.7 | 21.5 | 25.1 | 29.3 | 36.6 | 27.5 | 53.3  | 54.7 | 18.4                            | 21.3 | 24.3 | 24.7 | 28.1 | 31.1   | 60   |
| QuadsExp      | 17.7          | 14.3                 | 17.3 | 22.9 | 24.7 | 28.4 | 34.6 | 47.6 | 48.3 | 103.1 | 93.7 | 19.8                            | 21.7 | 28.4 | 31.7 | 41.9 | 48.5   | 60   |
| Regression    | 19.6          | 20.8                 | 21.7 | 18.7 | 18.1 | 18.6 | 21.7 | 23.0 | 16.1 | 27.5  | 26.5 | 19.8                            | 19.9 | 20.6 | 19.4 | 20.3 | 20.6   | 60   |
| NAIVE2        | 7.3           | 7.3                  | 9.9  | 10.1 | 13.5 | 11.4 | 13.3 | 13.1 | 13.3 | 10.0  | 29.8 | 10.2                            | 10.9 | 12.6 | 13.3 | 14.9 | 16.0   | 60   |
| D Mov.Averag  | 6.7           | 8.1                  | 11.3 | 14.9 | 18.2 | 18.8 | 21.3 | 23.3 | 14.8 | 26.5  | 30.5 | 13.2                            | 15.5 | 17.3 | 17.2 | 17.9 | 19.3   | 60   |
| D Sing Exp    | 7.1           | 6.1                  | 9.7  | 9.8  | 12.0 | 10.6 | 13.2 | 16.9 | 19.3 | 27.8  | 25.5 | 9.4                             | 10.2 | 11.8 | 12.3 | 13.6 | 14.9   | 60   |
| D ARR Exp     | 7.9           | 6.7                  | 10.3 | 9.4  | 12.5 | 10.9 | 13.0 | 16.0 | 12.2 | 27.0  | 24.9 | 9.9                             | 10.6 | 11.9 | 12.3 | 13.7 | 14.7   | 60   |
| D Holt Exp    | 6.9           | 6.0                  | 9.8  | 10.2 | 13.0 | 11.2 | 15.1 | 19.7 | 15.7 | 37.0  | 33.6 | 9.9                             | 11.0 | 13.1 | 14.2 | 16.5 | 18.4   | 60   |
| D Brown Exp   | 7.1           | 6.4                  | 9.5  | 9.9  | 12.2 | 10.5 | 14.8 | 19.6 | 15.4 | 38.7  | 35.6 | 9.5                             | 10.6 | 12.7 | 14.0 | 16.6 | 18.8   | 60   |
| D QuadsExp    | 7.0           | 6.1                  | 9.4  | 10.1 | 13.0 | 12.7 | 16.1 | 23.9 | 20.6 | 40.5  | 37.3 | 9.6                             | 11.2 | 14.1 | 16.1 | 19.0 | 21.4   | 60   |
| D Regression  | 10.1          | 10.9                 | 12.5 | 11.4 | 12.9 | 12.2 | 14.2 | 15.2 | 12.5 | 23.5  | 22.6 | 12.0                            | 12.4 | 13.0 | 12.8 | 13.8 | 14.6   | 60   |
| WINTERS       | 4.7           | 8.3                  | 9.8  | 10.5 | 13.6 | 11.5 | 14.7 | 19.9 | 14.9 | 34.0  | 32.7 | 10.7                            | 11.5 | 13.5 | 14.2 | 16.0 | 18.1   | 60   |
| Auton. AFP    | 11.1          | 10.3                 | 11.6 | 13.6 | 15.0 | 15.3 | 20.2 | 26.0 | 14.6 | 27.7  | 29.0 | 12.6                            | 14.3 | 17.0 | 16.4 | 17.4 | 18.6   | 60   |
| Bavesian F    | 11.1          | 8.1                  | 10.8 | 9.5  | 11.3 | 10.0 | 13.1 | 18.1 | 14.2 | 22.7  | 22.9 | 9.9                             | 10.5 | 11.9 | 12.4 | 13.5 | 14.4   | 60   |
| Comolininga A | 7.3           | 6.9                  | 9.4  | 10.0 | 12.4 | 10.8 | 14.5 | 18.6 | 12.7 | 31.1  | 28.7 | 9.7                             | 10.7 | 12.6 | 13.0 | 14.7 | 16.2   | 60   |
| Comolininga B | 6.9           | 7.1                  | 9.6  | 9.0  | 12.2 | 11.0 | 13.2 | 14.6 | 14.7 | 30.9  | 28.0 | 9.6                             | 10.4 | 12.3 | 13.2 | 15.1 | 16.5   | 60   |
| Box-Jenkins   | 4.4           | 10.5                 | 10.1 | 10.0 | 13.5 | 13.7 | 15.9 | 20.6 | 16.1 | 23.3  | 30.6 | 11.0                            | 12.3 | 14.3 | 14.8 | 15.8 | 17.2   | 60   |
| Levan-Jenski  | 10.1          | 10.8                 | 12.9 | 12.6 | 14.3 | 12.9 | 17.3 | 19.8 | 16.6 | 33.7  | 23.8 | 12.7                            | 13.5 | 14.7 | 14.9 | 16.9 | 17.6   | 60   |
| Parzen        | 8.5           | 11.0                 | 10.0 | 10.1 | 12.8 | 12.2 | 13.7 | 16.1 | 11.9 | 19.2  | 24.8 | 11.0                            | 11.6 | 12.7 | 12.9 | 13.6 | 14.9   | 60   |
| Average       | 10.9          | 10.7                 | 13.1 | 13.0 | 15.3 | 15.3 | 18.6 | 23.3 | 16.9 | 33.9  | 32.8 | 13.2                            | 14.4 | 16.3 | 16.6 | 18.5 | 20.0   |      |

Table 18. Average MAPE: non-seasonal data (51)









Table 26. Median APE: yearly data (20)

| METHODS      | MODEL<br>FITTING | FORECASTING HORIZONS |      |      |      |      |     | CUMULATIVE FORECASTING HORIZONS |     |     |     | N(MAX) |      |      |      |      |      |      |    |
|--------------|------------------|----------------------|------|------|------|------|-----|---------------------------------|-----|-----|-----|--------|------|------|------|------|------|------|----|
|              |                  | 1                    | 2    | 3    | 4    | 5    | 6   | 8                               | 12  | 15  | 18  | 1-4    | 1-6  | 1-8  | 1-12 | 1-15 | 1-18 | 1-20 |    |
| NAIVE 1      | 5.2              | 9.4                  | 12.0 | 17.1 | 20.6 | 22.5 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 10.2   | 12.9 | 14.9 | 12.9 | 12.9 | 12.9 | 12.9 | 20 |
| MOV.AVERAG   | 5.5              | 5.3                  | 9.2  | 17.1 | 20.6 | 24.1 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 10.2   | 12.9 | 14.9 | 12.9 | 12.9 | 12.9 | 12.9 | 20 |
| SINGLE EXP   | 5.5              | 4.9                  | 5.8  | 17.1 | 20.6 | 24.1 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 10.2   | 12.9 | 14.9 | 12.9 | 12.9 | 12.9 | 12.9 | 20 |
| ARR EXP      | 5.7              | 5.2                  | 10.3 | 13.3 | 21.4 | 27.7 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 12.5   | 15.1 | 15.1 | 15.1 | 15.1 | 15.1 | 15.1 | 20 |
| HOLT EXP     | 3.9              | 1.7                  | 2.5  | 3.3  | 7.0  | 11.7 | 7.5 | 0.0                             | 0.0 | 0.0 | 0.0 | 3.0    | 5.1  | 5.1  | 5.1  | 5.1  | 5.1  | 5.1  | 20 |
| BROWN EXP    | 3.3              | 2.6                  | 3.3  | 7.0  | 11.7 | 8.5  | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 4.0    | 6.3  | 6.3  | 6.3  | 6.3  | 6.3  | 6.3  | 20 |
| QUAD.EXP     | 3.3              | 3.0                  | 3.7  | 2.0  | 5.2  | 15.3 | 7.3 | 0.0                             | 0.0 | 0.0 | 0.0 | 3.9    | 5.3  | 5.3  | 5.3  | 5.3  | 5.3  | 5.3  | 20 |
| REGRESSION   | 4.7              | 6.1                  | 10.4 | 12.0 | 15.3 | 16.1 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 8.8    | 9.9  | 9.9  | 9.9  | 9.9  | 9.9  | 9.9  | 20 |
| NAIVE2       | 5.2              | 9.4                  | 12.0 | 17.1 | 20.6 | 22.5 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 10.2   | 12.9 | 14.9 | 12.9 | 12.9 | 12.9 | 12.9 | 20 |
| D MOV. AVERG | 5.5              | 5.3                  | 9.2  | 17.1 | 20.6 | 24.1 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 10.2   | 12.9 | 14.9 | 12.9 | 12.9 | 12.9 | 12.9 | 20 |
| SING EXP     | 5.5              | 4.9                  | 5.8  | 17.1 | 20.6 | 24.1 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 10.2   | 12.9 | 14.9 | 12.9 | 12.9 | 12.9 | 12.9 | 20 |
| ARR EXP      | 5.7              | 5.2                  | 10.3 | 13.3 | 21.4 | 27.7 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 12.5   | 15.1 | 15.1 | 15.1 | 15.1 | 15.1 | 15.1 | 20 |
| D HOLT EXP   | 3.5              | 1.7                  | 2.5  | 3.3  | 7.0  | 11.7 | 7.5 | 0.0                             | 0.0 | 0.0 | 0.0 | 4.0    | 6.3  | 6.3  | 6.3  | 6.3  | 6.3  | 6.3  | 20 |
| D BROWN EXP  | 3.3              | 2.6                  | 3.3  | 7.0  | 11.7 | 8.5  | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 3.9    | 5.3  | 5.3  | 5.3  | 5.3  | 5.3  | 5.3  | 20 |
| D QUAD EXP   | 3.3              | 3.0                  | 3.7  | 2.0  | 5.2  | 15.3 | 7.3 | 0.0                             | 0.0 | 0.0 | 0.0 | 4.0    | 6.3  | 6.3  | 6.3  | 6.3  | 6.3  | 6.3  | 20 |
| D REGRESS    | 4.7              | 6.1                  | 10.4 | 12.0 | 15.3 | 16.1 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 8.8    | 9.9  | 9.9  | 9.9  | 9.9  | 9.9  | 9.9  | 20 |
| PINTERS      | 3.5              | 1.7                  | 2.5  | 3.3  | 7.0  | 11.7 | 7.5 | 0.0                             | 0.0 | 0.0 | 0.0 | 4.0    | 6.3  | 6.3  | 6.3  | 6.3  | 6.3  | 6.3  | 20 |
| AUTOM. AEP   | 3.5              | 2.3                  | 3.3  | 3.3  | 6.4  | 17.5 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 3.9    | 5.3  | 5.3  | 5.3  | 5.3  | 5.3  | 5.3  | 20 |
| GAYESING F   | 3.3              | 3.3                  | 4.4  | 5.4  | 10.6 | 17.5 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 4.5    | 5.4  | 5.4  | 5.4  | 5.4  | 5.4  | 5.4  | 20 |
| COMBINING A  | 3.9              | 2.4                  | 4.3  | 6.4  | 19.7 | 13.2 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 5.7    | 7.7  | 7.7  | 7.7  | 7.7  | 7.7  | 7.7  | 20 |
| COMBINING B  | 4.1              | 3.3                  | 5.8  | 8.3  | 6.7  | 13.0 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 7.1    | 8.3  | 8.3  | 8.3  | 8.3  | 8.3  | 8.3  | 20 |
| BOX-JENKINS  | 0.0              | 2.8                  | 4.2  | 6.7  | 13.0 | 10.7 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 8.2    | 7.8  | 7.8  | 7.8  | 7.8  | 7.8  | 7.8  | 20 |
| LEWANDOWSKI  | 3.1              | 2.6                  | 5.3  | 8.9  | 11.2 | 11.5 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 4.8    | 6.2  | 6.2  | 6.2  | 6.2  | 6.2  | 6.2  | 20 |
| PARZEN       | 2.5              | 2.6                  | 5.3  | 6.2  | 11.2 | 11.5 | 0.0 | 0.0                             | 0.0 | 0.0 | 0.0 | 4.8    | 6.2  | 6.2  | 6.2  | 6.2  | 6.2  | 6.2  | 20 |

Table 27. Median APE: quarterly data (23)

| METHODS      | MODEL FITTING | FORECASTING HORIZONS |     |     |     |     |     |     |     | CUMULATIVE FORECASTING HORIZONS |     |     |     | N(MAX) |     |      |      |      |    |
|--------------|---------------|----------------------|-----|-----|-----|-----|-----|-----|-----|---------------------------------|-----|-----|-----|--------|-----|------|------|------|----|
|              |               | 1                    | 2   | 3   | 4   | 5   | 6   | 8   | 12  | 15                              | 18  | 1-4 | 1-6 |        | 1-8 | 1-12 | 1-15 | 1-18 |    |
| NAIVE 1      | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| MOV.AVERAG   | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| SINGLE EXP   | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| ARR EXP      | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| HOLT EXP     | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| BROWN EXP    | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| QUAD EXP     | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| REGRESSION   | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| NAIVE 2      | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| D MOV. AVERG | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| SING EXP     | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| ARR EXP      | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| HOLT EXP     | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| BROWN EXP    | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| QUAD EXP     | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| REGRESS      | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| PINTERS      | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| AUTOM. AEP   | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| GAYESING F   | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| COMBINING A  | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| COMBINING B  | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| BOX-JENKINS  | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| LEWANDOWSKI  | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |
| PARZEN       | 1.4           | 1.9                  | 1.2 | 1.4 | 1.6 | 1.3 | 1.3 | 1.3 | 1.3 | 1.3                             | 1.3 | 1.3 | 1.3 | 1.3    | 1.3 | 1.3  | 1.3  | 1.3  | 23 |

Table 28. Median APE: monthly data (68)

| METHODS      | MODEL FITTING |      | FORECASTING HORIZONS |      |      |      |      |      |      |      | CUMULATIVE FORECASTING HORIZONS |      |      |      |      |      |        |    |
|--------------|---------------|------|----------------------|------|------|------|------|------|------|------|---------------------------------|------|------|------|------|------|--------|----|
|              | 1             | 2    | 3                    | 4    | 5    | 6    | 8    | 12   | 15   | 18   | 1-4                             | 1-6  | 1-8  | 1-12 | 1-15 | 1-18 | N(MAX) |    |
| NAIVE 1      | 6.7           | 10.2 | 12.5                 | 9.6  | 11.0 | 11.1 | 11.6 | 10.3 | 7.3  | 18.1 | 18.3                            | 10.7 | 10.9 | 12.9 | 11.3 | 12.7 | 13.7   | 68 |
| MOV.AVERG    | 6.4           | 5.6  | 6.9                  | 9.4  | 8.6  | 8.6  | 9.0  | 10.1 | 11.9 | 19.0 | 19.6                            | 8.3  | 8.6  | 10.3 | 10.2 | 10.6 | 11.3   | 68 |
| SINGLE EXP   | 6.5           | 6.3  | 9.4                  | 9.4  | 10.0 | 9.5  | 10.1 | 11.5 | 11.7 | 17.1 | 19.5                            | 8.7  | 9.4  | 10.9 | 10.9 | 11.2 | 11.3   | 68 |
| ARR EXP      | 7.6           | 4.5  | 9.2                  | 7.6  | 11.0 | 9.1  | 11.7 | 11.5 | 12.1 | 15.5 | 16.0                            | 8.0  | 8.9  | 10.3 | 10.4 | 11.3 | 11.7   | 68 |
| HOLT EXP     | 6.2           | 6.1  | 11.2                 | 7.9  | 11.0 | 10.7 | 10.6 | 12.5 | 10.9 | 19.9 | 17.2                            | 8.9  | 10.0 | 10.6 | 11.1 | 12.3 | 12.9   | 68 |
| BROWN EXP    | 6.5           | 6.0  | 10.3                 | 8.8  | 11.0 | 10.3 | 11.3 | 11.6 | 12.1 | 21.0 | 25.3                            | 8.9  | 10.0 | 10.6 | 11.3 | 12.3 | 12.9   | 68 |
| QUAD. EXP    | 6.5           | 7.6  | 10.0                 | 11.1 | 11.4 | 10.6 | 11.0 | 16.0 | 12.3 | 8.8  | 16.1                            | 10.4 | 10.6 | 11.9 | 12.3 | 13.5 | 14.9   | 68 |
| REGRESSION   | 8.8           | 11.1 | 13.3                 | 11.1 | 12.6 | 12.8 | 14.6 | 13.0 | 12.3 | 17.9 | 15.7                            | 11.7 | 12.4 | 12.7 | 12.9 | 13.5 | 13.7   | 68 |
| NAIVE2       | 3.7           | 4.4  | 4.8                  | 6.2  | 7.9  | 5.8  | 6.1  | 9.9  | 7.3  | 12.9 | 15.6                            | 6.2  | 6.5  | 7.6  | 7.9  | 8.8  | 9.3    | 68 |
| D MOV.AVERG  | 3.3           | 6.5  | 8.0                  | 9.1  | 11.6 | 13.0 | 15.0 | 16.2 | 10.3 | 12.9 | 23.0                            | 8.4  | 9.7  | 11.6 | 11.6 | 12.3 | 13.4   | 68 |
| D SINGLE EXP | 3.4           | 2.7  | 5.8                  | 5.5  | 6.0  | 6.1  | 8.1  | 9.8  | 8.6  | 12.9 | 13.8                            | 5.4  | 5.8  | 6.9  | 7.3  | 7.9  | 8.6    | 68 |
| D ARR EXP    | 3.9           | 4.4  | 5.5                  | 6.1  | 6.6  | 6.7  | 6.8  | 9.6  | 7.9  | 12.7 | 13.7                            | 5.9  | 6.3  | 7.1  | 7.7  | 8.3  | 9.1    | 68 |
| D HOLT EXP   | 3.3           | 3.8  | 5.8                  | 7.3  | 7.6  | 5.4  | 8.1  | 9.9  | 7.4  | 12.7 | 15.0                            | 5.2  | 5.7  | 6.4  | 7.3  | 7.9  | 8.7    | 68 |
| D BROWN EXP  | 3.4           | 3.7  | 5.1                  | 6.9  | 5.6  | 6.2  | 6.3  | 9.4  | 9.7  | 16.2 | 20.1                            | 4.9  | 5.2  | 6.4  | 7.3  | 8.0  | 9.4    | 68 |
| D QUAD. EXP  | 3.4           | 6.0  | 7.6                  | 9.2  | 8.6  | 7.7  | 11.9 | 11.6 | 8.8  | 14.5 | 13.7                            | 8.3  | 8.7  | 9.4  | 10.1 | 10.7 | 10.7   | 68 |
| D REGRESS    | 3.4           | 6.0  | 6.6                  | 6.0  | 5.7  | 4.4  | 6.9  | 9.9  | 7.4  | 14.5 | 13.7                            | 8.3  | 8.7  | 9.4  | 10.1 | 10.7 | 10.7   | 68 |
| WINTERS      | 3.8           | 5.0  | 6.6                  | 7.8  | 6.6  | 6.5  | 6.5  | 9.5  | 9.2  | 13.7 | 15.2                            | 5.6  | 6.0  | 7.0  | 8.1  | 9.0  | 9.6    | 68 |
| AUTOM. AEP   | 4.4           | 5.0  | 6.6                  | 7.8  | 6.6  | 6.5  | 6.5  | 9.5  | 9.2  | 13.7 | 15.2                            | 5.6  | 6.0  | 7.0  | 8.1  | 9.0  | 9.6    | 68 |
| BAVESIAN F   | 5.3           | 5.0  | 6.6                  | 7.8  | 6.6  | 6.5  | 6.5  | 9.5  | 9.2  | 13.7 | 15.2                            | 5.6  | 6.0  | 7.0  | 8.1  | 9.0  | 9.6    | 68 |
| COMBINING A  | 3.2           | 3.0  | 5.9                  | 5.6  | 6.4  | 5.2  | 6.1  | 9.2  | 8.6  | 12.9 | 14.9                            | 5.5  | 5.5  | 6.8  | 7.0  | 7.6  | 8.3    | 68 |
| COMBINING B  | 3.3           | 3.0  | 5.9                  | 5.6  | 6.4  | 5.2  | 6.1  | 9.2  | 8.6  | 12.9 | 14.9                            | 5.5  | 5.5  | 6.8  | 7.0  | 7.6  | 8.3    | 68 |
| BOX-JENKINS  | 0.0           | 6.9  | 7.0                  | 6.8  | 5.4  | 5.7  | 7.8  | 9.1  | 8.6  | 12.4 | 16.4                            | 6.9  | 6.9  | 7.8  | 7.9  | 8.3  | 9.0    | 68 |
| LELANDOWSKI  | 4.0           | 5.7  | 5.6                  | 5.8  | 5.9  | 5.4  | 5.7  | 8.5  | 8.6  | 12.4 | 16.4                            | 6.9  | 6.9  | 7.8  | 7.9  | 8.3  | 9.0    | 68 |
| PARZEN       | 4.1           | 6.0  | 7.1                  | 7.3  | 8.3  | 6.7  | 7.6  | 8.3  | 6.6  | 11.5 | 11.6                            | 7.4  | 7.4  | 8.3  | 7.7  | 8.1  | 9.4    | 68 |

Table 29. Median APE: micro data (33)

| METHODS      | MODEL FITTING |      | FORECASTING HORIZONS |      |      |      |      |      |      |      | CUMULATIVE FORECASTING HORIZONS |      |      |      |      |      |        |    |
|--------------|---------------|------|----------------------|------|------|------|------|------|------|------|---------------------------------|------|------|------|------|------|--------|----|
|              | 1             | 2    | 3                    | 4    | 5    | 6    | 8    | 12   | 15   | 18   | 1-4                             | 1-6  | 1-8  | 1-12 | 1-15 | 1-18 | N(MAX) |    |
| NAIVE 1      | 9.0           | 13.5 | 20.4                 | 12.0 | 21.6 | 21.6 | 18.1 | 12.3 | 7.3  | 25.5 | 42.8                            | 18.2 | 18.2 | 18.1 | 18.9 | 18.7 | 18.7   | 33 |
| MOV.AVERG    | 8.7           | 19.7 | 11.5                 | 10.8 | 13.5 | 13.0 | 12.7 | 7.4  | 18.4 | 22.6 | 21.9                            | 11.5 | 11.7 | 11.6 | 11.8 | 12.4 | 12.5   | 33 |
| SINGLE EXP   | 8.8           | 11.7 | 11.5                 | 10.8 | 17.9 | 18.4 | 16.1 | 6.6  | 16.2 | 21.6 | 21.9                            | 11.5 | 11.7 | 11.6 | 11.8 | 12.4 | 12.5   | 33 |
| ARR EXP      | 9.7           | 13.5 | 15.9                 | 10.9 | 14.6 | 21.3 | 16.6 | 7.7  | 13.0 | 21.6 | 17.2                            | 13.6 | 14.0 | 13.8 | 13.8 | 13.9 | 13.9   | 33 |
| HOLT EXP     | 9.1           | 17.5 | 12.4                 | 12.0 | 10.4 | 22.4 | 10.9 | 10.6 | 12.1 | 23.7 | 11.5                            | 13.6 | 13.9 | 12.5 | 12.5 | 13.4 | 13.4   | 33 |
| BROWN EXP    | 9.1           | 13.0 | 13.0                 | 14.9 | 18.1 | 23.9 | 14.7 | 18.8 | 19.5 | 34.2 | 20.1                            | 13.6 | 15.0 | 15.1 | 15.1 | 15.6 | 15.6   | 33 |
| QUAD. EXP    | 8.6           | 11.4 | 12.5                 | 14.9 | 18.1 | 23.9 | 14.7 | 18.8 | 19.5 | 34.2 | 20.1                            | 13.6 | 15.0 | 15.1 | 15.1 | 15.6 | 15.6   | 33 |
| REGRESSION   | 10.8          | 11.1 | 9.9                  | 10.7 | 12.3 | 20.5 | 12.7 | 13.1 | 10.5 | 19.4 | 15.6                            | 10.6 | 11.7 | 14.0 | 11.7 | 12.2 | 12.2   | 33 |
| NAIVE2       | 9.9           | 9.3  | 10.7                 | 10.8 | 14.5 | 17.0 | 15.9 | 10.2 | 7.3  | 19.5 | 32.0                            | 12.9 | 13.5 | 10.6 | 10.6 | 11.3 | 11.3   | 33 |
| D MOV.AVERG  | 5.4           | 6.7  | 12.9                 | 9.7  | 15.6 | 9.3  | 12.4 | 10.2 | 12.8 | 23.0 | 18.6                            | 10.4 | 10.5 | 10.6 | 10.6 | 11.3 | 11.3   | 33 |
| D SINGLE EXP | 5.5           | 6.7  | 12.9                 | 9.7  | 15.6 | 9.3  | 12.4 | 10.2 | 12.8 | 23.0 | 18.6                            | 10.4 | 10.5 | 10.6 | 10.6 | 11.3 | 11.3   | 33 |
| D ARR EXP    | 5.1           | 7.5  | 12.5                 | 9.9  | 12.3 | 12.4 | 12.4 | 10.6 | 10.6 | 19.4 | 18.4                            | 10.4 | 10.4 | 10.9 | 10.4 | 10.6 | 10.6   | 33 |
| D HOLT EXP   | 5.1           | 7.5  | 12.5                 | 9.9  | 12.3 | 12.4 | 12.4 | 10.6 | 10.6 | 19.4 | 18.4                            | 10.4 | 10.4 | 10.9 | 10.4 | 10.6 | 10.6   | 33 |
| D BROWN EXP  | 5.2           | 11.0 | 10.9                 | 10.6 | 9.0  | 13.7 | 10.1 | 12.5 | 15.5 | 25.5 | 25.3                            | 10.5 | 10.8 | 10.3 | 11.3 | 11.3 | 11.3   | 33 |
| D QUAD. EXP  | 5.3           | 7.2  | 13.4                 | 11.6 | 13.0 | 13.0 | 11.6 | 10.6 | 7.3  | 15.0 | 16.8                            | 10.6 | 10.8 | 10.6 | 11.3 | 11.3 | 11.3   | 33 |
| D REGRESS    | 7.7           | 7.5  | 12.1                 | 11.6 | 12.7 | 12.8 | 11.3 | 10.6 | 12.5 | 18.0 | 16.8                            | 10.6 | 10.8 | 10.6 | 11.3 | 11.3 | 11.3   | 33 |
| WINTERS      | 5.8           | 6.2  | 13.1                 | 10.6 | 13.1 | 10.4 | 9.3  | 10.6 | 12.5 | 18.0 | 16.8                            | 10.6 | 10.8 | 10.6 | 11.3 | 11.3 | 11.3   | 33 |
| AUTOM. AEP   | 6.5           | 6.2  | 13.1                 | 10.6 | 13.1 | 10.4 | 9.3  | 10.6 | 12.5 | 18.0 | 16.8                            | 10.6 | 10.8 | 10.6 | 11.3 | 11.3 | 11.3   | 33 |
| BAVESIAN F   | 7.3           | 7.5  | 10.9                 | 11.7 | 11.8 | 14.2 | 10.6 | 10.6 | 5.3  | 16.9 | 17.1                            | 8.9  | 10.4 | 10.8 | 10.2 | 10.3 | 10.3   | 33 |
| COMBINING A  | 5.1           | 8.1  | 10.4                 | 9.7  | 15.0 | 12.9 | 11.4 | 9.3  | 5.3  | 16.9 | 17.1                            | 9.8  | 10.4 | 10.8 | 10.2 | 10.3 | 10.3   | 33 |
| COMBINING B  | 5.2           | 7.9  | 9.8                  | 8.7  | 15.0 | 12.9 | 11.4 | 9.3  | 5.3  | 16.9 | 17.1                            | 9.8  | 10.4 | 10.8 | 10.2 | 10.3 | 10.3   | 33 |
| BOX-JENKINS  | 0.0           | 10.7 | 12.6                 | 8.8  | 12.1 | 12.5 | 9.3  | 7.9  | 5.3  | 11.4 | 10.2                            | 11.3 | 11.7 | 10.9 | 10.4 | 11.2 | 11.2   | 33 |
| LELANDOWSKI  | 7.3           | 7.9  | 10.9                 | 8.8  | 12.1 | 12.5 | 9.3  | 7.9  | 5.3  | 11.4 | 10.2                            | 11.3 | 11.7 | 10.9 | 10.4 | 11.2 | 11.2   | 33 |
| PARZEN       | 6.4           | 10.7 | 13.2                 | 8.8  | 12.1 | 12.5 | 9.3  | 7.9  | 5.3  | 11.4 | 10.2                            | 11.3 | 11.7 | 10.9 | 10.4 | 11.2 | 11.2   | 33 |

Table 30. Median APE: macro data (35)

| METHODS     | MODEL FITTING | FORECASTING HORIZONS |     |     |      |      |      |      |      |      |      | CUMULATIVE FORECASTING HORIZONS |     |      |      | 1-18 | N(MAX) |
|-------------|---------------|----------------------|-----|-----|------|------|------|------|------|------|------|---------------------------------|-----|------|------|------|--------|
|             |               | 1                    | 2   | 3   | 4    | 5    | 6    | 8    | 12   | 15   | 18   | 1-4                             | 1-6 | 1-8  | 1-12 | 1-15 |        |
| NAIVE 1     | 2.5           | 3.0                  | 3.3 | 7.2 | 7.3  | 9.2  | 9.8  | 9.3  | 9.0  | 12.8 | 16.7 | 5.3                             | 7.1 | 7.1  | 9.2  | 9.8  | 9.1    |
| MOV.AVERAG  | 2.6           | 3.8                  | 6.6 | 7.0 | 7.5  | 9.0  | 11.1 | 9.9  | 11.3 | 12.9 | 14.3 | 5.6                             | 7.3 | 8.2  | 8.5  | 9.9  | 9.2    |
| SINGLE EXP  | 2.7           | 3.0                  | 5.8 | 7.3 | 7.3  | 9.0  | 10.1 | 9.9  | 11.7 | 12.9 | 14.1 | 5.5                             | 7.1 | 8.2  | 8.0  | 9.8  | 9.3    |
| ARR EXP     | 3.0           | 3.0                  | 4.4 | 6.3 | 7.0  | 9.0  | 11.7 | 8.5  | 11.2 | 12.9 | 15.0 | 5.8                             | 7.9 | 9.4  | 8.9  | 9.8  | 9.0    |
| HOLT EXP    | 2.1           | 2.5                  | 4.1 | 5.5 | 6.0  | 7.7  | 7.3  | 8.5  | 11.2 | 11.6 | 13.1 | 5.7                             | 4.9 | 5.8  | 6.2  | 7.3  | 7.5    |
| BROAD EXP   | 2.1           | 2.7                  | 3.7 | 4.8 | 5.7  | 7.6  | 9.0  | 9.3  | 12.1 | 11.0 | 11.9 | 5.7                             | 4.9 | 5.8  | 6.6  | 7.8  | 7.9    |
| QUAD EXP    | 2.1           | 2.7                  | 3.8 | 5.1 | 5.2  | 7.7  | 7.6  | 9.3  | 12.9 | 11.0 | 11.9 | 5.7                             | 4.9 | 5.8  | 6.6  | 7.8  | 7.9    |
| REGRESSION  | 4.7           | 6.6                  | 9.2 | 8.1 | 10.0 | 12.5 | 11.1 | 19.0 | 19.0 | 13.6 | 15.4 | 6.6                             | 9.1 | 10.7 | 11.0 | 12.5 | 9.3    |
| NAIVE2      | 1.8           | 2.0                  | 3.6 | 5.5 | 5.5  | 9.8  | 8.8  | 8.2  | 10.4 | 12.8 | 14.7 | 5.7                             | 8.5 | 10.2 | 11.0 | 12.5 | 9.8    |
| D MOV. AVG  | 1.7           | 2.2                  | 3.3 | 5.3 | 10.6 | 13.0 | 10.2 | 11.5 | 10.3 | 12.9 | 12.3 | 5.7                             | 7.7 | 9.2  | 9.3  | 9.8  | 9.8    |
| D SING EXP  | 1.8           | 2.2                  | 3.6 | 5.5 | 5.5  | 9.8  | 9.8  | 8.1  | 10.3 | 12.9 | 12.3 | 5.7                             | 7.7 | 9.2  | 9.3  | 9.8  | 9.8    |
| D ARR EXP   | 2.0           | 2.7                  | 4.5 | 5.5 | 5.5  | 9.8  | 10.7 | 9.4  | 10.4 | 10.1 | 18.0 | 2.1                             | 6.0 | 3.9  | 7.5  | 8.4  | 9.5    |
| D HOLT EXP  | 1.3           | 1.2                  | 1.8 | 3.0 | 3.1  | 5.1  | 6.7  | 7.2  | 9.8  | 7.7  | 6.3  | 2.7                             | 3.4 | 3.9  | 4.4  | 5.0  | 5.5    |
| D BRO=NEP   | 1.3           | 1.5                  | 1.8 | 3.2 | 4.0  | 5.6  | 7.8  | 8.9  | 9.8  | 7.7  | 14.0 | 2.7                             | 3.4 | 3.9  | 4.4  | 5.0  | 5.5    |
| D QUAD EXP  | 1.3           | 2.0                  | 3.8 | 5.5 | 5.5  | 10.7 | 12.5 | 13.0 | 10.4 | 13.6 | 12.7 | 2.7                             | 3.4 | 3.9  | 4.4  | 5.0  | 5.5    |
| D REGRES    | 3.8           | 5.0                  | 6.6 | 8.7 | 8.6  | 10.7 | 12.5 | 13.0 | 10.4 | 13.6 | 12.7 | 2.7                             | 3.4 | 3.9  | 4.4  | 5.0  | 5.5    |
| WINTERS     | 1.3           | 2.0                  | 3.1 | 3.2 | 3.5  | 5.6  | 6.7  | 8.9  | 9.8  | 7.7  | 6.3  | 2.7                             | 3.4 | 3.9  | 4.4  | 5.0  | 5.5    |
| AYTON, AEP  | 1.5           | 1.5                  | 2.2 | 3.2 | 4.0  | 5.6  | 6.6  | 9.2  | 10.9 | 12.8 | 10.8 | 2.7                             | 3.4 | 3.9  | 4.4  | 5.0  | 5.5    |
| BAYESIAN F  | 2.1           | 2.2                  | 2.7 | 4.6 | 5.3  | 5.3  | 8.7  | 10.5 | 10.7 | 12.8 | 10.2 | 2.7                             | 3.4 | 3.9  | 4.4  | 5.0  | 5.5    |
| COMBINING A | 1.4           | 1.6                  | 2.9 | 3.4 | 5.8  | 5.7  | 5.7  | 5.7  | 9.3  | 12.4 | 11.5 | 2.7                             | 3.4 | 3.9  | 4.4  | 5.0  | 5.5    |
| COMBINING B | 1.5           | 1.6                  | 2.9 | 3.4 | 5.8  | 5.7  | 5.7  | 5.7  | 9.3  | 12.4 | 11.5 | 2.7                             | 3.4 | 3.9  | 4.4  | 5.0  | 5.5    |
| BOX-JENKINS | 0.0           | 1.7                  | 2.8 | 3.0 | 5.2  | 4.9  | 7.8  | 8.2  | 10.0 | 18.4 | 11.8 | 2.9                             | 4.2 | 4.2  | 4.9  | 5.8  | 6.8    |
| LEWANDOWSKI | 2.1           | 3.5                  | 3.6 | 4.2 | 5.2  | 5.0  | 7.8  | 8.2  | 10.0 | 18.4 | 11.8 | 2.9                             | 4.2 | 4.2  | 4.9  | 5.8  | 6.8    |
| PARZEN      | 1.3           | 1.8                  | 2.0 | 2.4 | 5.1  | 5.7  | 5.7  | 6.7  | 6.3  | 18.1 | 11.0 | 2.2                             | 3.4 | 4.2  | 4.8  | 5.0  | 5.3    |

Table 31. Median APE: seasonal data (60)

| METHODS     | MODEL FITTING | FORECASTING HORIZONS |      |      |      |      |      |      |      |      |      | CUMULATIVE FORECASTING HORIZONS |      |      |      | 1-18 | N(MAX) |
|-------------|---------------|----------------------|------|------|------|------|------|------|------|------|------|---------------------------------|------|------|------|------|--------|
|             |               | 1                    | 2    | 3    | 4    | 5    | 6    | 8    | 12   | 15   | 18   | 1-4                             | 1-6  | 1-8  | 1-12 | 1-15 |        |
| NAIVE 1     | 9.0           | 12.5                 | 10.6 | 10.0 | 12.0 | 13.0 | 13.3 | 10.3 | 6.9  | 18.9 | 18.7 | 11.0                            | 12.9 | 13.9 | 13.9 | 13.9 | 9.0    |
| MOV.AVERAG  | 8.2           | 9.8                  | 10.5 | 10.0 | 10.7 | 10.4 | 11.3 | 10.3 | 11.7 | 18.1 | 18.7 | 11.0                            | 12.9 | 13.9 | 13.9 | 13.9 | 9.0    |
| SINGLE EXP  | 7.6           | 9.1                  | 10.0 | 10.7 | 10.4 | 11.3 | 10.3 | 10.3 | 11.7 | 18.1 | 18.7 | 11.0                            | 12.9 | 13.9 | 13.9 | 13.9 | 9.0    |
| ARR EXP     | 8.9           | 11.0                 | 10.5 | 11.7 | 10.4 | 11.3 | 10.3 | 10.3 | 11.7 | 18.1 | 18.7 | 11.0                            | 12.9 | 13.9 | 13.9 | 13.9 | 9.0    |
| HOLT EXP    | 7.1           | 6.7                  | 9.0  | 11.5 | 11.3 | 11.3 | 11.4 | 10.3 | 10.3 | 13.6 | 13.7 | 10.6                            | 11.4 | 11.4 | 12.0 | 12.0 | 9.0    |
| BROAD EXP   | 7.7           | 11.2                 | 10.5 | 11.1 | 11.8 | 11.3 | 11.4 | 10.3 | 10.3 | 13.6 | 13.7 | 10.6                            | 11.4 | 11.4 | 12.0 | 12.0 | 9.0    |
| QUAD EXP    | 7.5           | 11.2                 | 10.5 | 11.1 | 11.8 | 11.3 | 11.4 | 10.3 | 10.3 | 13.6 | 13.7 | 10.6                            | 11.4 | 11.4 | 12.0 | 12.0 | 9.0    |
| REGRESSION  | 3.8           | 4.5                  | 5.9  | 12.0 | 12.5 | 16.5 | 18.3 | 19.3 | 9.8  | 13.6 | 13.7 | 5.6                             | 10.3 | 10.3 | 12.0 | 12.0 | 9.0    |
| NAIVE2      | 3.4           | 4.5                  | 5.9  | 12.0 | 12.5 | 16.5 | 18.3 | 19.3 | 9.8  | 13.6 | 13.7 | 5.6                             | 10.3 | 10.3 | 12.0 | 12.0 | 9.0    |
| D MOV. AVG  | 3.5           | 4.5                  | 5.9  | 12.0 | 12.5 | 16.5 | 18.3 | 19.3 | 9.8  | 13.6 | 13.7 | 5.6                             | 10.3 | 10.3 | 12.0 | 12.0 | 9.0    |
| D SING EXP  | 3.5           | 4.5                  | 5.9  | 12.0 | 12.5 | 16.5 | 18.3 | 19.3 | 9.8  | 13.6 | 13.7 | 5.6                             | 10.3 | 10.3 | 12.0 | 12.0 | 9.0    |
| D ARR EXP   | 4.0           | 4.5                  | 5.3  | 7.7  | 6.0  | 6.1  | 6.3  | 9.1  | 6.5  | 12.2 | 13.2 | 5.1                             | 5.4  | 5.4  | 7.1  | 8.0  | 9.0    |
| D HOLT EXP  | 3.4           | 3.6                  | 5.3  | 7.7  | 6.0  | 6.1  | 6.3  | 9.1  | 6.5  | 12.2 | 13.2 | 5.1                             | 5.4  | 5.4  | 7.1  | 8.0  | 9.0    |
| D BRO=NEP   | 3.4           | 3.6                  | 5.3  | 7.7  | 6.0  | 6.1  | 6.3  | 9.1  | 6.5  | 12.2 | 13.2 | 5.1                             | 5.4  | 5.4  | 7.1  | 8.0  | 9.0    |
| D QUAD EXP  | 3.4           | 3.6                  | 5.3  | 7.7  | 6.0  | 6.1  | 6.3  | 9.1  | 6.5  | 12.2 | 13.2 | 5.1                             | 5.4  | 5.4  | 7.1  | 8.0  | 9.0    |
| D REGRES    | 5.1           | 5.3                  | 5.1  | 7.5  | 7.5  | 9.5  | 9.5  | 10.0 | 5.7  | 10.4 | 10.3 | 6.4                             | 6.6  | 6.3  | 7.1  | 7.9  | 9.0    |
| WINTERS     | 5.0           | 5.3                  | 5.1  | 7.5  | 7.5  | 9.5  | 9.5  | 10.0 | 5.7  | 10.4 | 10.3 | 6.4                             | 6.6  | 6.3  | 7.1  | 7.9  | 9.0    |
| AYTON, AEP  | 5.0           | 5.3                  | 5.1  | 7.5  | 7.5  | 9.5  | 9.5  | 10.0 | 5.7  | 10.4 | 10.3 | 6.4                             | 6.6  | 6.3  | 7.1  | 7.9  | 9.0    |
| BAYESIAN F  | 5.0           | 5.3                  | 5.1  | 7.5  | 7.5  | 9.5  | 9.5  | 10.0 | 5.7  | 10.4 | 10.3 | 6.4                             | 6.6  | 6.3  | 7.1  | 7.9  | 9.0    |
| COMBINING A | 3.5           | 3.1                  | 5.8  | 5.2  | 7.6  | 5.6  | 6.6  | 11.2 | 5.3  | 12.6 | 13.1 | 5.1                             | 5.4  | 5.1  | 6.2  | 6.8  | 9.0    |
| COMBINING B | 3.5           | 3.1                  | 5.8  | 5.2  | 7.6  | 5.6  | 6.6  | 11.2 | 5.3  | 12.6 | 13.1 | 5.1                             | 5.4  | 5.1  | 6.2  | 6.8  | 9.0    |
| BOX-JENKINS | 3.0           | 3.1                  | 5.8  | 5.2  | 7.6  | 5.6  | 6.6  | 11.2 | 5.3  | 12.6 | 13.1 | 5.1                             | 5.4  | 5.1  | 6.2  | 6.8  | 9.0    |
| LEWANDOWSKI | 3.0           | 3.1                  | 5.8  | 5.2  | 7.6  | 5.6  | 6.6  | 11.2 | 5.3  | 12.6 | 13.1 | 5.1                             | 5.4  | 5.1  | 6.2  | 6.8  | 9.0    |
| PARZEN      | 4.4           | 6.0                  | 6.3  | 6.3  | 7.6  | 7.2  | 7.6  | 8.1  | 6.2  | 11.4 | 11.8 | 7.0                             | 7.2  | 7.2  | 7.2  | 8.0  | 9.0    |

Table 32. Median APE: Non-seasonal data (51)

| METHODS     | MODEL<br>FITTING | FORECASTING HORIZONS |     |      |      |      |      |      |      |      |      | CUMULATIVE FORECASTING HORIZONS |      |      |      |      | N(MAX) |
|-------------|------------------|----------------------|-----|------|------|------|------|------|------|------|------|---------------------------------|------|------|------|------|--------|
|             |                  | 1                    | 2   | 3    | 4    | 5    | 6    | 8    | 12   | 15   | 18   | 104                             | 106  | 108  | 1012 | 1015 | 1018   |
| NAIVE 1     | 3.0              | 4.0                  | 5.6 | 10.8 | 13.9 | 13.1 | 16.5 | 10.8 | 15.3 | 12.8 | 22.8 | 7.6                             | 8.7  | 10.1 | 10.1 | 10.3 | 10.9   |
| MOV-AVERG   | 2.8              | 3.2                  | 5.8 | 10.8 | 14.5 | 16.1 | 17.1 | 11.2 | 15.1 | 12.5 | 22.7 | 7.6                             | 10.8 | 11.4 | 11.4 | 11.4 | 12.0   |
| SINGLE EXP  | 3.0              | 4.0                  | 5.8 | 10.8 | 14.5 | 16.1 | 16.5 | 11.2 | 15.1 | 12.5 | 22.7 | 7.6                             | 10.8 | 11.4 | 11.4 | 11.4 | 12.0   |
| ARR EXP     | 3.4              | 4.8                  | 6.3 | 9.4  | 15.2 | 17.4 | 20.6 | 11.2 | 11.4 | 12.5 | 22.7 | 8.7                             | 10.8 | 10.8 | 10.8 | 10.8 | 10.8   |
| MOLT EXP    | 2.4              | 3.5                  | 4.5 | 6.0  | 12.3 | 12.3 | 14.2 | 11.8 | 12.3 | 16.3 | 17.9 | 5.5                             | 7.5  | 9.5  | 10.5 | 11.5 | 11.5   |
| BROWN EXP   | 2.4              | 3.5                  | 4.5 | 6.0  | 12.3 | 12.3 | 14.2 | 11.8 | 12.3 | 16.3 | 17.9 | 5.5                             | 7.5  | 9.5  | 10.5 | 11.5 | 11.5   |
| QUAD EXP    | 2.3              | 3.5                  | 4.5 | 6.0  | 12.3 | 12.3 | 14.2 | 11.8 | 12.3 | 16.3 | 17.9 | 5.5                             | 7.5  | 9.5  | 10.5 | 11.5 | 11.5   |
| REGRESSION  | 5.4              | 6.5                  | 8.5 | 10.7 | 13.5 | 17.3 | 17.3 | 19.0 | 14.3 | 13.8 | 25.3 | 10.1                            | 15.3 | 15.3 | 15.3 | 15.3 | 15.3   |
| NAIVE2      | 3.0              | 4.0                  | 5.8 | 10.8 | 14.5 | 16.1 | 17.1 | 11.2 | 15.1 | 12.5 | 22.7 | 7.6                             | 10.8 | 11.4 | 11.4 | 11.4 | 12.0   |
| MOV-AVRG    | 2.8              | 3.2                  | 5.8 | 10.8 | 14.5 | 16.1 | 17.1 | 11.2 | 15.1 | 12.5 | 22.7 | 7.6                             | 10.8 | 11.4 | 11.4 | 11.4 | 12.0   |
| SING EXP    | 3.0              | 4.0                  | 5.8 | 10.8 | 14.5 | 16.1 | 16.5 | 11.2 | 15.1 | 12.5 | 22.7 | 7.6                             | 10.8 | 11.4 | 11.4 | 11.4 | 12.0   |
| ARR EXP     | 3.4              | 4.8                  | 6.3 | 9.4  | 15.2 | 17.4 | 20.6 | 11.2 | 11.4 | 12.5 | 22.7 | 8.7                             | 10.8 | 10.8 | 10.8 | 10.8 | 10.8   |
| MOLT EXP    | 2.4              | 3.5                  | 4.5 | 6.0  | 12.3 | 12.3 | 14.2 | 11.8 | 12.3 | 16.3 | 17.9 | 5.5                             | 7.5  | 9.5  | 10.5 | 11.5 | 11.5   |
| BROWN EXP   | 2.4              | 3.5                  | 4.5 | 6.0  | 12.3 | 12.3 | 14.2 | 11.8 | 12.3 | 16.3 | 17.9 | 5.5                             | 7.5  | 9.5  | 10.5 | 11.5 | 11.5   |
| QUAD EXP    | 2.3              | 3.5                  | 4.5 | 6.0  | 12.3 | 12.3 | 14.2 | 11.8 | 12.3 | 16.3 | 17.9 | 5.5                             | 7.5  | 9.5  | 10.5 | 11.5 | 11.5   |
| REGRESS     | 6.4              | 8.5                  | 9.5 | 10.7 | 13.5 | 17.3 | 17.3 | 19.0 | 14.3 | 13.8 | 25.3 | 10.1                            | 15.3 | 15.3 | 15.3 | 15.3 | 15.3   |
| VINTENS     | 2.4              | 3.4                  | 4.5 | 6.0  | 12.3 | 12.3 | 14.2 | 11.8 | 12.3 | 16.3 | 17.9 | 5.5                             | 7.5  | 9.5  | 10.5 | 11.5 | 11.5   |
| AUTOM. AEP  | 2.2              | 2.4                  | 4.5 | 7.3  | 9.2  | 10.9 | 8.4  | 10.9 | 14.3 | 17.8 | 20.5 | 5.5                             | 6.8  | 7.3  | 8.3  | 9.4  | 10.3   |
| BAYESIAN F  | 3.6              | 2.6                  | 4.4 | 5.3  | 7.9  | 12.2 | 10.8 | 10.3 | 18.3 | 17.6 | 21.5 | 5.5                             | 7.3  | 7.3  | 8.3  | 9.4  | 10.3   |
| COMBINING A | 2.4              | 2.6                  | 4.3 | 6.0  | 9.3  | 15.0 | 12.0 | 12.4 | 11.3 | 12.4 | 23.9 | 6.0                             | 7.0  | 7.0  | 8.1  | 9.1  | 10.0   |
| COMBINING B | 2.4              | 2.6                  | 4.3 | 6.0  | 9.3  | 15.0 | 12.0 | 12.4 | 11.3 | 12.4 | 23.9 | 6.0                             | 7.0  | 7.0  | 8.1  | 9.1  | 10.0   |
| BOX-JENKINS | 0.0              | 2.0                  | 4.2 | 5.2  | 8.1  | 8.9  | 10.7 | 10.3 | 11.0 | 18.2 | 14.1 | 4.4                             | 5.4  | 7.2  | 7.9  | 9.7  | 10.8   |
| LEWANDOWSKI | 3.7              | 4.2                  | 4.8 | 4.8  | 7.2  | 7.9  | 10.6 | 10.3 | 5.1  | 17.2 | 11.5 | 5.0                             | 6.0  | 5.4  | 5.4  | 5.4  | 5.4    |
| PARZEN      | 2.1              | 2.6                  | 4.7 | 5.3  | 7.2  | 9.2  | 11.6 | 13.9 | 7.8  | 18.1 | 11.5 | 5.0                             | 6.0  | 5.4  | 5.4  | 5.4  | 5.4    |

Combining can be profitably used to reduce forecasting errors by simply averaging the predictions of a few forecasting methods.

### Significant differences

Are the differences in the relative performance of the various methods, discussed in the previous section, statistically significant? It is not easy to test statistically each of the statements presented in the previous section for two reasons. First, the errors are non-symmetric, which excludes using parametric statistics. Second, not enough data are available to test differences in subcategories. This is particularly true when the 111 series are used. However, several of the statements made in the previous paragraphs can be substantiated by statistical tests.

Assuming normality in the errors (an assumption which does not hold true), an analysis of various methods can be performed to test for statistically significant differences of how well these methods forecast the different series used, the various horizons and, overall, both series and horizons. These three aspects will be called series, horizons and methods respectively for which tests have been conducted by using a straightforward analysis of variance approach. Table 33

Table 33. Analysis of variance for different groupings of methods

| Grouping of Methods     | F-Tests and Degrees of Freedom | Type of Data |            |              |             |            |              |              |              |               |
|-------------------------|--------------------------------|--------------|------------|--------------|-------------|------------|--------------|--------------|--------------|---------------|
|                         |                                | Yearly       |            |              | Quarterly   |            |              | Monthly      |              |               |
|                         |                                | Methods      | Horizons   | Series       | Methods     | Horizons   | Series       | Methods      | Horizons     | Series        |
| 24 Methods <sup>1</sup> | F-Test                         | 7.73         | 31.18      | 256          | 4.36        | 49.69      | 119          | 10.45        | 12.11        | 293           |
|                         | D.F.                           | 23<br>2879   | 5<br>2879  | 19<br>2879   | 23<br>4415  | 7<br>4415  | 22<br>4415   | 23<br>29375  | 17<br>29375  | 67<br>29375   |
| 21 Methods <sup>2</sup> | F-Test                         | 12.84        | 34.7       | 263          | 13.51       | 8.16       | 41.39        | 4.81         | 13.37        | 418           |
|                         | D.F.                           | 20<br>22679  | 5<br>22679 | 179<br>22679 | 20<br>34103 | 5<br>34103 | 262<br>34103 | 20<br>233225 | 17<br>233225 | 616<br>233225 |
| 8 Methods <sup>3</sup>  | F-Test                         | 3.33         | 10.74      | 91.48        | 3.87        | 11.46      | 114          | 1.37<br>**   | 4.83         | 101.9         |
|                         | D.F.                           | 7<br>959     | 5<br>959   | 19<br>959    | 7<br>1471   | 7<br>1471  | 22<br>1471   | 7<br>9791    | 17<br>9791   | 67<br>9791    |
| 5 Methods <sup>4</sup>  | F-Test                         | 12.6         | 7.76       | 44           | 7.75        | 2.29<br>** | 17.86        | 5.75         | 4.64         | 131           |
|                         | D.F.                           | 4<br>5177    | 5<br>5177  | 178<br>5177  | 4<br>7901   | 7<br>7901  | 4<br>1901    | 4<br>54887   | 17<br>54887  | 616<br>54887  |

All differences except those with \*\* are significantly different than zero, at least at a 99% level.

<sup>1</sup> For a list of the 24 methods see Table 1 (a).

<sup>2</sup> For a list of the 21 methods see Table 1 (b).

<sup>3</sup> The 8 methods are: Deseasonalized single exponential smoothing, Holt, Winters, Automatic AEP, Bayesian Forecasting, Box-Jenkins, Lewandowski, and Parzen. These methods are considered to be the group containing the best methods, varying the least among themselves, for the 111 series.

<sup>4</sup> The 5 methods are: Deseasonalized single exponential smoothing, Holt, Winters, Automatic AEP, and Bayesian forecasting. These methods are considered to be the group containing the best methods, varying the least among themselves, for all the 1001 series.

shows the *F*-tests together with the corresponding degrees of freedom. The great majority of the *F*-tests are significant at the 1 % level. In general, variations due to series are much more significant than those due to horizons which in turn are more significant than those due to methods.

In order to perform the analysis of variance, the various methods were subdivided into four different groupings. The first grouping included all 24 methods (111 series), the second grouping the 21 methods for which all 1001 series have been used. However, comparisons involving all methods may be meaningless because some methods (e.g. simple methods when the data have not been deseasonalized) were only used as a yardstick to judge the relative performance of the remaining methods. Thus, a third grouping of the eight most accurate methods was used, and a last grouping of five of these eight methods for which all 1001 series were available was also made. Table 33 presents the *F*-tests and gives the degrees of freedom for each of the four groupings.

Table 34 is more appropriate for the accuracy data in this study. It is a non-parametric multiple comparisons procedure for the average rankings (Hollander and Wolfe, 1973). Those differences in average rankings, which are statistically significant at the 1 % level, need to be bigger than the corresponding value shown in the last row of Tables 34(a) and 34(b). The base method for comparison was the deseasonalized single exponential smoothing. None of the differences in the

Table 34(a). Differences in overall (i.e. periods 1-18) average rankings from deseasonalized single exponential smoothing and corresponding value of *d*-statistic (1001 series)

| Methods            | All Data | Yearly Data | Quarterly Data | Monthly Data | Micro Data | Macro Data | Industry Data | Demographic Data | Seasonal Data | Non-Seasonal Data |
|--------------------|----------|-------------|----------------|--------------|------------|------------|---------------|------------------|---------------|-------------------|
| D. Holt Exp.       | -.01     | 1.04*       | .27*           | -.15*        | -.12*      | .38*       | -.13          | -.34*            | -.03          | .01               |
| Winters            | .02      | 1.04*       | .26*           | -.11*        | -.14*      | .40*       | -.10          | -.20*            | .03           | .01               |
| Automatic AEP      | -.08     | 1.10*       | .30*           | -.25*        | -.32*      | .26*       | -.23*         | .08              | -.24*         | .15*              |
| Bayesian Forecast. | -.09     | .75*        | -.13           | -.17*        | -.14*      | .18*       | -.33*         | -.08             | -.12*         | .05               |
| d-Statistic        | .23      | .27         | .22            | .09          | .12        | .12        | .14           | .18              | .09           | .09               |

\* Denotes significant differences at a 1% level.

Table 34(b). Differences in overall (i.e. periods 1-18) average rankings from deseasonalized single exponential smoothing and corresponding value of *d*-statistic (111 series)

| Methods            | All Data | Yearly Data | Quarterly Data | Monthly Data | Micro Data | Macro Data | Industry Data | Demographic Data | Seasonal Data | Non-Seasonal Data |
|--------------------|----------|-------------|----------------|--------------|------------|------------|---------------|------------------|---------------|-------------------|
| D. Holt Exp.       | .23      | 2.22*       | .18            | .04          | -.07       | 1.14*      | -.03          | -.50             | .09           | .50*              |
| Winters            | .17      | 2.22*       | .19            | -.04         | -.42       | 1.01*      | .07           | -.06             | -.01          | .50*              |
| Automatic AEP      | -.04     | 1.59*       | .29            | -.25         | -.63*      | .73*       | -.09          | -.23             | -.21          | .27               |
| Bayesian Forecast. | -.05     | 1.38*       | -.97           | -.05         | -.61*      | .56*       | -.14          | .16              | -.12          | .08               |
| Box-Jenkins        | -.09     | 1.13        | .39            | -.06         | -.19       | .37        | .01           | .28              | -.07          | .36               |
| Lewandoski         | -.31     | 1.62*       | .08            | .22          | .70*       | .78*       | -.30          | -.34             | -.11          | 1.08*             |
| Parzen             | -.20     | 1.57*       | .88            | -.04         | -.03       | .82*       | -.15          | .14              | .01           | .56*              |
| d-Statistic        | .36      | 1.27        | 1.02           | .40          | .57        | .55        | .63           | .82              | .42           | .46               |

\* Denotes significant differences at a 1% level.

average rankings are statistically significant as far as *all* of the data are concerned. This is true for each of the forecasting horizons *and* the average of all forecasts. However, the differences become significant when subcategories of data are used, which shows that there is not one single method which can be used across the board indiscriminately. The forecasting user should be selective. It is interesting to note that in Table 34(a) *all* differences in yearly, quarterly, monthly, micro and macro data are significant.

Furthermore, note that the signs in yearly, quarterly and monthly data are positive (meaning that the corresponding methods perform statistically better than deseasonalized single exponential smoothing) whereas for monthly and micro all the signs are negative.

Finally, fewer significant differences exist in Table 34(b) because there are only 111 data for the comparisons. However, the signs (with a few exceptions—e.g. Lewandowski) and the statistically significant values follow a pattern similar to that of Table 34(a). The implications of the results shown in Tables 34(a) and 34(b) are highly important as far as the practical utilization of extrapolative methods is concerned.

### Non-significant differences

A most interesting aspect of making the comparisons has been those differences which turned out to be statistically non-significant. These cases are listed below and it is hoped that future research will explain the reasons why this is happening and what are the implications for forecasting.

1. It was expected that forecasts before 1974 would be more accurate than those after 1974. In fact, when the data were separated into two corresponding categories, no significant difference, in post-sample forecasting accuracy, was found between pre and post 1974 data. Similarly, when the data were separated into a category which ended during or just before a recession and another including all other series, the differences between the two categories were not found to be statistically significant.
2. The parameters of the various models were found by minimizing the one-step-ahead Mean Square Error for each of the series involved. All forecasts are therefore one-step-ahead forecasts. When the method required more values in order to obtain additional forecasts, the forecasts already found were used for this purpose. In addition to this one-step-ahead forecast, multiple lead time forecasts were also obtained for the deseasonalized Holt method. That is, optimal parameters for 1, 2, 3, ..., 18 periods ahead were obtained and a single forecast was found, using these optimal parameters. Thus, for monthly data, each series was re-run 18 times, each time obtaining optimal parameters and one *L*-period ahead forecasts. For the method used to obtain multiple lead time forecasts, no significant differences were observed between their accuracy and that of one-period forecasts.
3. Several variations of Winters' Exponential Smoothing were run but no significant differences from the specific Holt-Winters model used in this paper were observed.
4. Two variations of Adaptive Response Rate Exponential Smoothing (ARRES) were run. The one which used a delay in the adaptation of alpha did not produce significantly more accurate forecasts than the non-delayed version. Furthermore, ARRES did not perform better than non-adaptive exponential smoothing methods, a finding consistent with that of Gardner and Dannenbring (1980).
5. In addition to deseasonalizing the data by a simple ratio-to-moving average (centred) decomposition method, the same deseasonalization was also done
  - (a) by using the seasonal indices obtained by the CENSUS II method;
  - (b) by using the one-year-ahead forecast of the seasonal factors obtained by the CENSUS II method.



Neither of these deseasonalized procedures produced forecasts which were better than those of the ratio to centred moving average method reported in Naive 2.

6. It makes little difference as to what method to use for industry-wide series, when there are demographic series, or for data that exhibit seasonality.
7. Finally, some preliminary work concerning the effect of the number of data points on accuracy has not produced evidence that, as the number of data points increases, relative performance is improved. This finding is consistent with that found in Makridakis and Hibon (1979) and raises some interesting questions about the length of time series to be used in forecasting.

## CONCLUSIONS

The major purpose of this paper has been to summarize the results of a forecasting competition of major extrapolation (time series) methods and look at the different factors affecting forecasting accuracy. If the forecasting user can discriminate in his choice of methods depending upon the type of data (yearly, quarterly, monthly), the type of series (macro, micro, etc.) and the time horizon of forecasting, then he or she could do considerably better than using a single method across all situations—assuming, of course, that the results of the present study can be generalized. Overall, there are considerable gains to be made in forecasting accuracy by being selective (e.g. see Tables 34(a) and 34(b)). Furthermore, combining the forecasts of a few methods improves overall forecasting accuracy over and above that of the individual forecasting methods used in the combining.

The question that deserves further consideration is obviously this: why do some methods do better than others under various conditions? This could not be attributed simply to chance, given the large number of series used. Even though further research will be necessary to provide us with more specific reasons as to why this is happening, a hypothesis may be advanced at this point, stating that statistically sophisticated methods do not do better than simple methods (such as deseasonalized exponential smoothing) when there is considerable randomness in the data. This is clear with monthly and micro data in which randomness is much more important than in quarterly or yearly macro data. Finally, it seems that seasonal patterns can be predicted equally well by both simple and statistically sophisticated methods. This is so, it is believed, because of the instability of seasonal variations that dominate the remaining of the patterns and which can be forecasted as accurately by averaging seasonality as in using any statistically sophisticated approach.

The authors of this paper hope that the information presented will help those interested in forecasting to understand better the factors affecting forecasting accuracy and realize the differences that exist among extrapolative (time series) forecasting methods.

## APPENDIX I

### The accuracy measures

This appendix presents the various accuracy measures used in the competition.

Two sets of errors were calculated for each method. The first was arrived at by fitting a model to the first  $n - m$  values (where  $m = 6$  for yearly, 8 for quarterly, and 18 for monthly data) of each of the series and calculating the error  $e_t$  as follows:

$$e_t = X_t - \hat{X}_t \quad (1)$$

where  $X_t$  is the actual value, and  $\hat{X}_t$  is one-period-ahead forecasted value.

Two so-called errors of 'model fitting' were also calculated as follows, where all summations go from 1 to  $n - m$ :

(a) The mean percentage error (MAPE) =  $(n - m)^{-1} \sum (|e_i|/X_i)(100)$  (2)

(b) The mean square error (MSE) =  $(n - m)^{-1} \sum e_i^2$ . (3)

(c) The percentage of time the error for method  $i$  was smaller than that for method  $j$  was also recorded.

(d) The ranking of each method in relation to all others. (The best method received the ranking of 1, the second of 2, the third of 3 and so forth.) The rankings were then averaged for all series.

(e) The median absolute percentage error.

The second set of errors involves the last  $m$  values, which were utilized as *post-sample* measures to determine the magnitude of the errors. The two measurements shown in equations (2) and (3), as well as the percentage of time method  $i$  was better than method  $j$ , and the average rankings were also computed for up to  $m$  forecasting horizons, starting at period  $n - m + 1$ . In addition, the median of the absolute percentage error was computed.

In no instance have the last  $m$  values been used to develop a forecasting model or estimate its parameters. The model fitting *always* involved only the first  $n - m$  values for each series.

## APPENDIX 2

### The methods

(1) Naive 1.

$$\text{Model fitting: } \hat{X}_{t+1} = X_t, \quad (4)$$

where  $t = 1, 2, 3, \dots, n - m$

$$\text{Forecasts: } \hat{X}_{n-m+k} = X_{n-m}, \quad (5)$$

where  $k = 1, 2, 3, \dots, m$ .

(2) Simple moving average.

$$\text{Model fitting: } \hat{X}_{t+1} = \frac{X_t + X_{t-1} + X_{t-2} + \dots + X_{t-N+1}}{N}, \quad (6)$$

where  $N$  is chosen so as to minimize  $\sum e_t^2$ , again summing over  $t$  from 1 to  $n - m$

$$\text{Forecasts: } \hat{X}_{n-m+k} = \frac{X_{n-m+k-1} + X_{n-m+k-2} + \dots + X_{n-m+k-N}}{N}. \quad (7)$$

When the subscript of  $X$  on the right-hand side of (7) is larger than  $n - m$ , the corresponding forecasted value is substituted.

(3) Single exponential smoothing.

$$\text{Model fitting: } \hat{X}_{t+1} = \alpha X_t + (1 - \alpha)\hat{X}_t, \quad (8)$$

where  $\alpha$  is chosen so as to minimise  $\sum e_t^2$ , the mean square error where again summing is over  $t$  from 1 to  $n - m$ .

$$\text{Forecasts: } \hat{X}_{n-m+k} = \alpha X_{n-m} + (1 - \alpha)\hat{X}_{n-m+k-1}. \quad (9)$$

## (4) Adaptive response rate exponential smoothing.

The equations are exactly the same in (8) and (9), except  $\alpha$  varies with  $t$ . The value of  $\alpha_t$  is found by

$$\alpha_t = |E_t/M_t|. \quad (10)$$

where  $E_t = \beta c_t + (1 - \beta)E_{t-1}$  and  $M_t = \beta|c_t| + (1 - \beta)M_{t-1}$ .  $\beta$  is set at 0.2.

## (5) Holt's two-parameter linear exponential smoothing.

$$\text{Model fitting: } S_t = \alpha X_t + (1 - \alpha)(S_{t-1} + T_{t-1}). \quad (11)$$

$$T_t = \beta(S_t - S_{t-1}) + (1 - \beta)T_{t-1}. \quad (12)$$

$$\hat{X}_{t+1} = S_t + T_t. \quad (13)$$

The values of  $\alpha$  and  $\beta$  are chosen so as to minimize the mean square error. This was achieved by a complete search of all possibilities.

$$\text{Forecasts: } \hat{X}_{n-m+k} = S_{n-m} + T_{n-m}(k). \quad (14)$$

## (6) Brown's one-parameter linear exponential smoothing.

$$\text{Model fitting: } S'_t = \alpha X_t + (1 - \alpha)S'_{t-1}, S''_t = \alpha S'_t + (1 - \alpha)S''_{t-1}, \hat{X}_{t+1} = a_t + b_t. \quad (15)$$

where  $a_t = 2S'_t - S''_t$  and  $b_t = (1 - \alpha)^{-1}(S'_t - S''_t)$ .

The value of  $\alpha$  is chosen so as to minimize the mean square error.

$$\text{Forecasts: } \hat{X}_{n-m+k} = a_{n-m} + b_{n-m}(k). \quad (16)$$

## (7) Brown's one-parameter quadratic exponential smoothing.

$$\text{Model fitting: } S'_t = \alpha X_t + (1 - \alpha)S'_{t-1}, \quad (17)$$

$$S''_t = \alpha S'_t + (1 - \alpha)S''_{t-1}, \quad (18)$$

$$S'''_t = \alpha S''_t + (1 - \alpha)S'''_{t-1}, \quad (19)$$

$$\hat{X}_{t+1} = a_t + b_t + 1/2c_t, \quad (20)$$

where

$$a_t = 3S'_t - 3S''_t + S'''_t, \quad b_t = \alpha \{2(1 - \alpha)^2\}^{-1} \{ (6 - 5\alpha)S'_t - (10 - 8\alpha)S''_t + (4 - 3\alpha)S'''_t \}$$

and

$$c_t = \alpha(1 - \alpha)^{-2}(S'_t - 2S''_t + S'''_t)$$

The value of  $\alpha$  is chosen so as to minimize the mean square error.

$$\text{Forecasts: } \hat{X}_{n-m+k} = a_{n-m+k} + b_{n-m+k}(k) + 1/2c_{n-m+k}(k)^2. \quad (21)$$

## (8) Linear regression trend fitting.

$$\text{Model fitting: } \hat{X}_t = a + bt, \quad (22)$$

where  $t = 1, 2, 3, \dots, n - m$ , and  $a$  and  $b$  are chosen so as to minimize the sum of the square errors by solving the normal equations:

$$a = \frac{\sum X}{n - m} - b \frac{\sum t}{n - m}, \quad b = \frac{(n - m) \sum tX - t \sum X}{(n - m) \sum t^2}.$$

where all summations go from 1 to  $n - m$

$$\text{Forecasts: } \hat{X}_{n-m+k} = a + b(n - m + k). \quad (23)$$

(9) Naive 2 as Naive 1 (see (1)) but the data are deseasonalized and then seasonalized.

The seasonal indices for deseasonalizing and seasonalizing the data were done by the decomposition method of the ratio-to-moving averages. The specifics of this method can be seen in Makridakis and Wheelwright (1978, pp. 94-100).

(10) Deseasonalized single moving average as in (2) except the data have been deseasonalized and then reseasonalized.

(11) Deseasonalized single exponential smoothing as in (3) except for deseasonalizing.

(12) Deseasonalized adaptive response rate exponential smoothing as in (4) except for deseasonalizing.

(13) Deseasonalized Holt's exponential smoothing as in (5) except for deseasonalizing.

(14) Deseasonalized Brown's linear exponential smoothing as in (6) except for deseasonalizing.

(15) Deseasonalized Brown's quadratic exponential smoothing as in (7) except for deseasonalizing.

(16) Deseasonalized linear regression as in (8) except for deseasonalizing.

The deseasonalizing of the various methods (9) to (16) was done by computing seasonal indices with a simple ratio-to-moving average (centred) decomposition method. The  $n - m$  data of each series were first adjusted to seasonality, as

$$X'_t = X_t / S_j$$

where  $X'_t$  is the seasonally adjusted (deseasonalized) value and  $S_j$  is the corresponding seasonal index for period  $t$ .

The forecasts for  $\hat{X}'_{n-m-1}, \hat{X}'_{n-m-2}, \dots, \hat{X}'_n$  were reseasonalized as:

$$\hat{X}_{n-m+k} = \hat{X}'_{n-m+k}(S_j)$$

(17) Holt-Winters' linear and seasonal exponential smoothing.

If the data have no seasonality (i.e. significantly different to zero autocorrelation coefficient at lag 4, for quarterly data, or at a lag 12, for yearly data) then Holt's exponential smoothing is used (see (5) above). Otherwise, Winters' three-parameter model is used:

Model fitting:

$$\begin{aligned} S_t &= \alpha \frac{X_t}{I_{t-L}} + (1 - \alpha)(S_{t-1} + T_{t-1}), \\ T_t &= \gamma(S_t - S_{t-1}) + (1 - \gamma)T_{t-1}, \\ I_t &= \beta \frac{X_t}{S_t} + (1 - \beta)I_{t-L}, \\ \hat{X}_{t+1} &= (S_t + T_t)I_{t-L+1}, \end{aligned} \quad (24)$$

where  $L$  is the length of seasonality.

The values of  $\alpha$ ,  $\beta$  and  $\gamma$  were chosen so as to minimize the MSE. This was done by a complete search of all possibilities, using a grid search method.

$$\text{Forecasts: } \hat{X}_{n-m+k} = (S_{n-12} + kT_{n-12})I_{n-12+k}. \quad (25)$$

Initial values for all exponential smoothing methods were computed by backforecasting on the data. This was done in order to eliminate any possible disadvantage of the exponential smoothing methods.

(18) AEP (Automatic) Carbone-Longini.<sup>1</sup>

The Carbone-Longini filtered method (1977) was developed to provide a practical solution to the problem of adapting over time parameters of mixed additive and multiplicative models without *a priori* information. The general model formulation to which the method applies is written as:

$$y(t) = \left[ \left( \sum_{i=1}^n a_i(t) z_i(t) \right) \sum_{j=1}^p b_j(t) x_j(t) \right] + e(t)$$

where, for time  $t$ ,  $y(t)$  is the value of a dependent variable;  $z_i(t)$  denotes the value assigned to the qualitative dimension  $i$  (1 if observed, 0 if not);  $x_j(t)$  denotes the measurement of the quantitative feature  $j$ ;  $a_i(t)$  and  $b_j(t)$  are the corresponding parameters at time  $t$ ; and  $e(t)$  is an undefined error term. In time series analysis the  $z_i(t)$  could represent, for example, seasons (months, quarters, etc.), and the  $x_j(t)$ , different lag values of a time series.

A negative damped feedback mechanism is used to adapt the parameters over time. It consists of the following two simple recursive formulae:

$$b_j(t) = b_j(t-1) + |b_j(t-1)| \left[ \frac{y(t) - \hat{y}(t)}{|\hat{y}(t)|} \cdot \frac{x_j(t)}{\bar{x}_j(t)} \cdot \mu \right]$$

$$a_i(t) = a_i(t-1) + a_i(t-1) \left[ \frac{y(t) - \hat{y}(t)}{|\hat{y}(t)|} \cdot z_i(t) \cdot \mu l \right]$$

where  $\hat{y}(t)$  is a forecast of  $y(t)$  computed on the basis of the parameters at time  $t-1$ ;  $\bar{x}_j(t) = s x_j(t) + (1-s) x_j(t-1)$  with  $0 < s < 1$ ;  $\mu$  is a damping factor between 0 and 1; and  $l < 1$  is a positive constant for all  $t$ .

In this study, the method was applied under the most naive of assumptions (see Bretschneider, Carbone and Longini (1979)) in an automatic execution mode with no user intervention. For the 1001 series, the model formulation was one in which all  $z_i(t)$  were assumed to be 0 and  $x(t)$  represented lag values (autoregressors) of a time series. In other words, the model reduced to an autoregressive equation with time dependent parameters. Of the information available (series names, country, seasonal indicator and type of data), only the type of data (yearly, quarterly or monthly) was used. The number of autoregressive variables was at least 3, 4 or 12 for yearly, quarterly or monthly data respectively. The exact number for a specific series was established automatically as well as the data transformation applied (difference transformation when necessary) by internal program decision rules (automatic analysis of sample autocorrelation functions). In all cases, an identical initialization procedure was applied. Initial values of the parameters were set to the inverse of the number of autoregressors in a model. Start up values for the exponential smoothing means were always 100 with smoothing constant equal to 0.01. A damping factor of 0.06 was applied in all cases. Finally, the necessary learning process (iterating several times forward/backward through the data) was stopped by an internal program decision rule. A discussion of the internal decision rules can be found in Carbone (1980).

The results were obtained in a single run (around three (3) hours of CPU time on a IBM 370/158). Most of the computer time was devoted to reading and writing and report generation. The work could have been efficiently performed on a 64K micro-processor. Again, no revisions of forecasts through personalized analysis were performed.

## (19) Bayesian Forecasting.

At its simplest, Bayesian Forecasting is merely a particular method of model estimation in which

<sup>1</sup> Carbone expresses his thanks to Serge Nadeau for his help in designing the AEP package which was specifically used for this study.

a prior probability distribution is assigned to the model's parameters and these are subsequently updated as new data become available to produce forecasts. (In the U.K., in particular, the term has recently become synonymous with the approach developed by Harrison and Stevens (1971, 1976). A program developed by Stevens has been used in this study.)

The basic transformed Bayesian forecasting model is:

$$Z_t = \mu_t + S_{i,t} + \varepsilon_t \quad \varepsilon_t \sim N(0, V_\varepsilon)$$

$$\mu_t = \mu_{t-1} + \beta_t + \delta\mu_t \quad S\mu_t \sim N(0, V_\mu)$$

$$\beta_t = \beta_{t-1} + \delta\beta_t \quad \delta\beta_t \sim N(0, V_\beta)$$

$$S_{i,t} = S_{i,t-1} + \delta S_{i,t} \quad i = 1, 2, \dots, \tau$$

where  $Z_t = \log Y_t$ , and  $\mu_t$ ,  $\beta_t$  and  $S_t$  are the log transforms of the 'level', 'trend' and 'seasonal' factors.

In matrix notation these equations may be written

$$Z_t = X_t \theta_t + v_t; v_t \sim N(0, V_t) \text{—the observation equation}$$

$$\theta_t = G \theta_{t-1} + w_t; w_t \sim N(0, W_t) \text{—the systems equation}$$

$$\theta_t = (\mu_t, \beta_t, S_{1,t}, \dots, S_{\tau,t})$$

For non-seasonal data,  $X_t = (1, 0, 0, \dots, 0, 0, 0, \dots, 0)$

$$G = \begin{bmatrix} 1 & 1 \\ 0 & 1 & 0 \\ 0 & & 1 \end{bmatrix}$$

For seasonal data,  $X_t = (1, 0, 0, \dots, 1, 0, 0, \dots, 0)$ .

The model ( $M$ ) is characterized by the four matrices  $M \equiv (X_t, G, V_t, W_t)$ . With these matrices assumed known it is possible to estimate  $\theta_{t+k}$  and  $Z_{t+k}$  using the Kalman Filter to produce the  $k$ -period-ahead forecast.

The Bayesian model employed in the forecasting competition is the so-called multi-state model. Here it is supposed that in each and every period the process is in one of a number of possible states  $M^{(i)}; i = 1, \dots, 4$ ; the stable or no change state, the step change, the slope change and the transient. It is assumed that these states occur randomly over time with constant probability of occurrence independent of the previous state of the process.

These four different states can be characterized by the following set of parameters:

| Model type               | Prior weight | RE( $i$ )       | RG( $i$ )     | RD( $i$ )   | RS( $i$ )          |
|--------------------------|--------------|-----------------|---------------|-------------|--------------------|
| $M^{(1)}$ : no change    | 1000         | 0               | 0             | 0           | $\sqrt{(12/\tau)}$ |
| $M^{(2)}$ : step change  | 10           | 0               | 30 % of level | 0           | 0                  |
| $M^{(3)}$ : slope change | 10           | 0               | 0             | 12.5 % p.a. | 0                  |
| $M^{(4)}$ : transient    | 100          | 33.3 % of level | 0             | 0           | 0                  |

where  $\tau$  denotes periodicity.

The variance of the raw observations is assumed to be  $C^2(EY_t)^2$ . The parameter  $C$  is estimated by defining a range of values within which it could lie. These were selected by individual examination of each of the 1001 series. With each new datapoint the posterior probability of each  $C$  value being correct is calculated. The estimated value of  $C$  is merely the average of the eleven  $C$  values weighted

by their respective probabilities. This average value is then used to calculate the posterior probability distribution of  $\theta_{t+k}$  and  $Z_{t+k}$ .

Bayesian forecasting is iterative in the sense that starting with a subjectively specified prior for the mean level of the series, the growth and seasonal factors, a set of forecasts can be produced. A new observation is then used to update the priors and generate new forecasts.

The priors used to start off the process were

|                                    | Low     | Mean  | High  |
|------------------------------------|---------|-------|-------|
| Prior level (units per period)     | 1       | 1000  | 10 %  |
| Prior growth (% p.a.)              | -33.3 % | 0     | 50 %  |
| Prior seasonality (if appropriate) | 50 %    | 100 % | 200 % |

Note that, as no fitting takes place, the entries in the various tables under 'model fitting' have no meaning for the method of Bayesian forecasting.

#### (20) Combining Forecasts (Combining A).

This method uses the simple average of methods (11), (12), (13), (14), (17), and (18).

$$\text{Model fitting: } \hat{X}_t = \frac{\hat{X}_t^{(11)} + \hat{X}_t^{(12)} + \hat{X}_t^{(13)} + \hat{X}_t^{(14)} + \hat{X}_t^{(17)} + \hat{X}_t^{(18)}}{6},$$

where  $t = 1, \dots, n-m$  and  $\hat{X}_t^{(i)}$  is  $\hat{X}_t$  for method (i).

$$\text{Forecasts: } \hat{X}_{n-m+k} = \frac{\hat{X}_{n-m+k}^{(11)} + \hat{X}_{n-m+k}^{(12)} + \hat{X}_{n-m+k}^{(13)} + \hat{X}_{n-m+k}^{(14)} + \hat{X}_{n-m+k}^{(17)} + \hat{X}_{n-m+k}^{(18)}}{6},$$

where  $k = 1, \dots, m$ .

#### (21) Combining Forecasts (Combining B).

Here a weighted average of the six methods used in (19) is used. The weights are based on the sample covariance matrix of percentage errors for these six methods for the model fitting for each series.

$$\text{Model Fitting: } \hat{X}_t = \sum_i w_i \hat{X}_t^{(i)},$$

with

$$w_i = \sum_j x_{ij} / \sum_h \sum_j x_{hj},$$

where all summations are over the set {11, 12, 13, 14, 17, 18} and the  $d_{ij}$  terms are elements of the inverse of the covariance matrix of percentage errors. That is, if  $S = (\beta_{ij})$ , where

$$\beta_{ij} = \sum_{t=1}^{n-m} [u_t^{(i)} - \bar{u}^{(i)}][u_t^{(j)} - \bar{u}^{(j)}] / (n-m)$$

and

$$u_t^{(i)} = e_t^{(i)} / X_t = [X_t - \hat{X}_t^{(i)}] / X_t \quad \text{and} \quad \bar{u}^{(i)} = \sum_{t=1}^{n-m} u_t^{(i)} / (n-m),$$

then  $d_{ij}$  is the element in row  $i$  and column  $j$  of  $S^{-1}$ .

$$\text{Forecasts: } \hat{X}_{n-m+k} = \sum_i w_i \hat{X}_{n-m+k}^{(i)},$$

where  $k = 1, \dots, m$  and the summation is over the set {11, 12, 13, 14, 17, 18}.

## (22) The Box-Jenkins Methodology.

The Box-Jenkins technique has become very popular since the publication of their book in 1970. In general, the process consists of a cycle of four components: data transformation, model identification, parameter estimation and diagnostic checking. Only after the diagnostic checks indicate that an adequate model has been constructed, are the forecasts produced. The methodology is well documented (see for example Box and Jenkins (1970), Granger and Newbold (1977), Nelson (1973), Anderson (1976) or Chatfield and Prothero (1973)) so that here we give only the broad outlines of what was done. The only data transformation considered was the natural logarithm, which was applied when there appeared to be an exponential trend, or heteroskedasticity in the errors. To look at wider classes of transformation appeared to be too expensive. Model identification was via the autocorrelation function in particular with the partial autocorrelation function used for confirming evidence, combined with a rigorously imposed 'Principle of Parsimonious Parameterization'. Once a tentative model had been identified, the parameters, together with a mean or trend constant were estimated. Diagnostic checking consisted of an examination of the 'important' lag residual autocorrelations and the original Box-Pierce  $X^2$  statistic, together with limited overfitting. To produce the forecasts, the model was extrapolated, together with a correction factor applied, if the logarithms had been analysed.

Finally, the projections were examined to see if they seemed reasonable in light of the historic data. This last check was used mainly to distinguish between competing adequate models.

## (23) Lewandowski's FORSYS System.

$X_t$ , the time series, is decomposed as follows:

$$X_t = M_t S_t + c_t \quad (26)$$

The mean,  $M_t$ , is defined by a moving average process which is basically of exponential smoothing type. For instance, for a linear model,  $M_t$  is defined as:

$$M_t = 2(M1_t) - M2_t \quad (27)$$

where

$$M1_t = \sum \frac{X_{t-\theta}}{S_{t-\theta}} \alpha_{t-\theta} \prod (1 - \alpha_{t-\theta})^\theta \quad (28)$$

$$M2_t = \sum M1_{t-\theta} \alpha_{t-\theta} \prod (1 - \alpha_{t-\theta})^\theta \quad (29)$$

The smoothing constant  $\alpha_t$  is given by:

$$\alpha_t = \alpha_0 + \Delta\alpha_t$$

The values of  $\alpha_t$  vary as follows:

$$\alpha_0 = \alpha_0 \rho^{f_1(\sigma_t^{(1)})}$$

$$\Delta\alpha_t = \kappa_0 \rho^{f_2(\sigma_t^{(2)})} - \kappa_1 \rho^{f_3(\Sigma_t^*)}$$

where  $\sigma_t^{(1)}$  is a measure of the stability of the series and is defined as:

$$\sigma_t^{(1)} = \left| \frac{\text{MAD}_t}{M_t} \right|$$

and where

$$\text{MAD}_t = |\varepsilon_t| \gamma + (1 - \gamma) \text{MAD}_{t-1}$$



$\sigma_t^{(2)}$  is a normalized measure of the randomness of the series. It is defined as:

$$\sigma_t^{(2)} = \left| \frac{\varepsilon_t}{\text{MAD}_t} \right|$$

and finally,  $\Sigma_t^*$  is a tracking signal defined as follows:

$$\Sigma_t^* = \frac{\Sigma_t}{\text{MAD}_t}$$

where

$$\Sigma_t = \Sigma_{t-1}(1 - \gamma_{S_t}) + \varepsilon_t$$

where  $\gamma_{S_t}$  can be thought of as the coefficient of decay, that is:

$$\gamma_{S_t} = \gamma_{S_0} [1 - \rho^{f_s(\sigma_t^{(2)})}]$$

The seasonal coefficients are found by an exponential smoothing process similar to that of (28) and (29) which is:

$$S_t = \sum \frac{X_{t-\tau}}{M_{t-\tau}} \beta_{t-\tau} \prod (1 - \beta_{t-\tau})$$

where

$$\beta_t = \beta_0 \tilde{\rho}^{f_s(\Sigma_t^*)}$$

The forecasting of the series is given by combining the components of (1), that is  $M_t$  and  $S_t$ . This results in the following projections:

$$\hat{X}_{t+k}^{(1)} = M(\alpha)_t + \kappa T(\alpha)_t + \kappa^2 Q(\alpha)_t$$

$$\hat{X}_{t+k}^{(s)} = M(a^s)_t + \kappa T(a^s)_t + \kappa^2 Q(a^s)_t$$

$$\hat{X}_{t+k}^{(2)} = M(\alpha^*)_t + \kappa T(\alpha^*)_t$$

Finally, the forecasts are found by

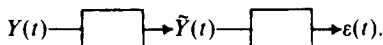
$$\hat{X}_{t+k} = \{ \hat{X}_{t+k}^{(s)} \delta_{t+k} \} S_{t+k}$$

For more details, see Lewandowski (1979).

#### (24) ARARMA Methodology.

The models used are called ARARMA models (see Parzen (1979), (1980)) because the model computed adaptively for a time series is based on sophisticated time series analysis of ARMA schemes (a short memory model) fitted to residuals of simple extrapolation (a long memory model obtained by parsimonious 'best lag' non-stationary autoregression).

The model fitted to a time series  $Y(\cdot)$  is an iterated model



If needed to transform a long memory series  $Y$  to a short memory series  $\tilde{Y}$ ,  $\tilde{Y}(t)$  is chosen to satisfy one of the three forms

$$\tilde{Y}(t) = Y(t) - \hat{\phi}(\hat{\tau})Y(t - \hat{\tau}),$$

$$\tilde{Y}(t) = Y(t) - \phi_1 Y(t-1) - \phi_2 Y(t-2), \quad (30)$$

$$\tilde{Y}(t) = Y(t) - \phi_1 Y(t - \tau - 1) - \phi_2 Y(t - \tau) \quad (31)$$

Usually  $\hat{Y}(t)$  is short memory, then it is transformed to a white noise, or no memory, time series  $\varepsilon(t)$  by an approximating autoregressive scheme  $AR(\hat{m})$  whose order  $\hat{m}$  is chosen by an order determining criterion (called CAT).

To determine the best lag  $\hat{\tau}$ , a non-stationary autoregression is used; either a maximum lag  $M$  is fixed and  $\hat{\tau}$  is chosen as the lag minimizing over all  $\tau$

$$\sum_{t=M+1}^T \{Y(t) - \phi(\tau)Y(t-\tau)\}^2$$

or  $\hat{\tau}$  is chosen as the lag minimizing over all  $\tau$

$$\sum_{t=\tau+1}^T \{Y(t) - \phi(\tau)Y(t-\tau)\}^2$$

For each  $\tau$ , one determines  $\phi(\tau)$ , and then one determines  $\hat{\tau}$  (the optimal value of  $\tau$ ) as the value minimizing

$$\text{Err}(\tau) = \sum_{t=M+1}^T \{Y(t) - \hat{\phi}(\tau)Y(t-\tau)\}^2$$

or

$$\text{Err}(\tau) = \sum_{t=\tau+1}^T \{Y(t) - \hat{\phi}(\tau)Y(t-\tau)\}^2$$

The decision as to whether the time series is long memory or not is based on the value of  $\text{Err}(\hat{\tau})$ . An *ad hoc* rule is used if  $\text{Err}(\hat{\tau}) < 8/T$ , the time series is considered long memory. When this criterion fails one often seeks transformations of the form of (30) or (31), using semi-automatic rules described elsewhere (see Parzen (1982)).

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